Four methods for analyzing partial interval recording data, with application to single-case research

James E. Pustejovsky & Daniel M. Swan
The University of Texas at Austin
Contact: pusto@austin.utexas.edu

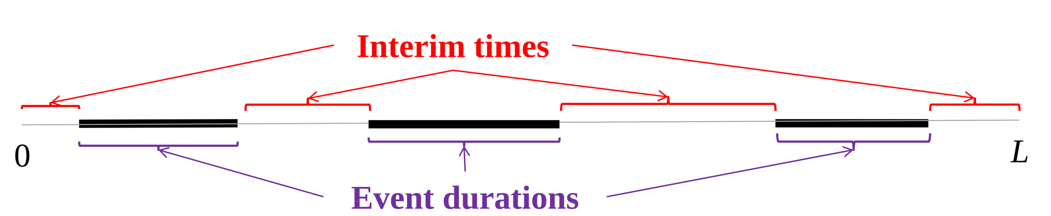
Abstract

Partial interval recording is a procedure for collecting measurements based on direct observation of behavior. It is used in several areas of educational and psychological research, particularly in connection with single-case research. Measurements collected using partial interval recording suffer from construct invalidity because they are not readily interpretable in terms of the underlying properties of the behavior. Using an alternating renewal process model for the behavior under observation, we demonstrate that ignoring the construct invalidity of PIR data can produce misleading inferences, such as inferring that an intervention reduces the prevalence of an undesirable behavior when in fact it has the opposite effect. We then propose four different methods for analyzing PIR summary measurements, all of which produce estimates of interpretable behavioral parameters.

Background & Goals

- Direct observation of behavior is used extensively in certain areas of education and psychological research. It is considered a hallmark of single-case research.
- Aspects of behavior that are often of interest include:
  - Prevalence: the proportion of time that a behavior occurs;
  - Incidence: the rate at which behavioral events occur.
- Procedures for recording observations include continuous recording, event counting, momentary time sampling, partial interval recording (PIR).
- PIR measures neither prevalence nor incidence (Altman, 1974; Kraemer, 1979; Mann et al., 1991), yet remains in wide use within single-case reseach (Mudford, Taylor, & Martin, 2009; Rapp et al., 2007).
- Few analytic methods for PIR data account for its construct invalidity.
  - Altmann & Wagner (1970) suggest complementary-log-log transformation, but this is only valid under Poisson process.
  - Other methods (Suen & Ary, 1986; Quera, 1990; Pustejovsky, 2013) require access to within-session data rather than just summary measurements.
- Our goal is to develop methods for analyzing PIR data that are framed in terms of underlying behavioral characteristics and that account for the construct invalidity of the measurements.

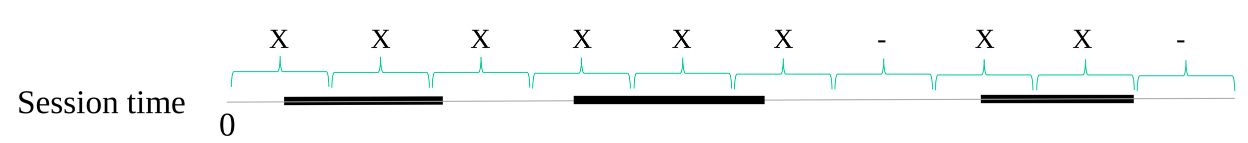
The behavior stream



Partial Interval Recording (PIR)

- Divide session into K intervals, each of length c.
- For each interval, observer records whether behavior occurred ***at any point*** during the interval.
- Calculate the proportion of intervals with behavior:

Y = (# Intervals with behavior) / K



Common modeling assumptions

Consider a study in which there are two samples of PIR measurements, of size n_0 and n_1, respectively. Let Y_{si} denote the measurement from session i in sample s. We model the behavior stream by an equilibrium alternating renewal process. Specifically, we assume that:

1. Event durations from session i in sample s are iid draws from a distribution with mean μ_s and cumulative distribution $F_s(x; \mu_s)$.
2. Interim times from session i in sample s are iid draws from a distribution with mean λ_s and cumulative distribution $G_s(x; \lambda_s)$.
3. Event durations and interim times are mutually independent.
4. The process is aperiodic and in equilibrium.

Under these assumptions:

- $\phi_s = \mu_s / (\mu_s + \lambda_s)$ is the prevalence of the behavior in sample s.
- $\zeta_s = 1 / (\mu_s + \lambda_s)$ is the incidence of the behavior in sample s.
- The expected value of a PIR measurement is

$$E\left(Y_{si}\right) = \phi_s + \zeta_s \int_0^c 1 - G_s\left(x; \lambda_s\right) dx$$

Target parameters

The goal is to compare behavioral characteristics between the two samples. We focus on comparisons in the form of log-ratios:

- Log-prevalence ratio: $\ln(\phi_1 / \phi_0)$
- Log-incidence ratio: $\ln(\zeta_1 / \zeta_0)$
- Log-interim ratio: $\ln(\lambda_0 / \lambda_1)$

Statistics & notation

Sample mean: $\bar{y}_s$

Sample variance: s_s^2

$$\tilde{y}_s = \begin{cases} 1/\left(n_s K\right) & \text{if } \quad \bar{y}_s = 0 \\ \bar{y}_s & \text{if } \ 0 < \bar{y}_s < 1 \\ 1 - 1/\left(n_s K\right) & \text{if } \quad \bar{y}_s = 1 \end{cases}$$

Log response ratio: $R = \ln\left(\tilde{y}_1\right) - \ln\left(\tilde{y}_0\right)$

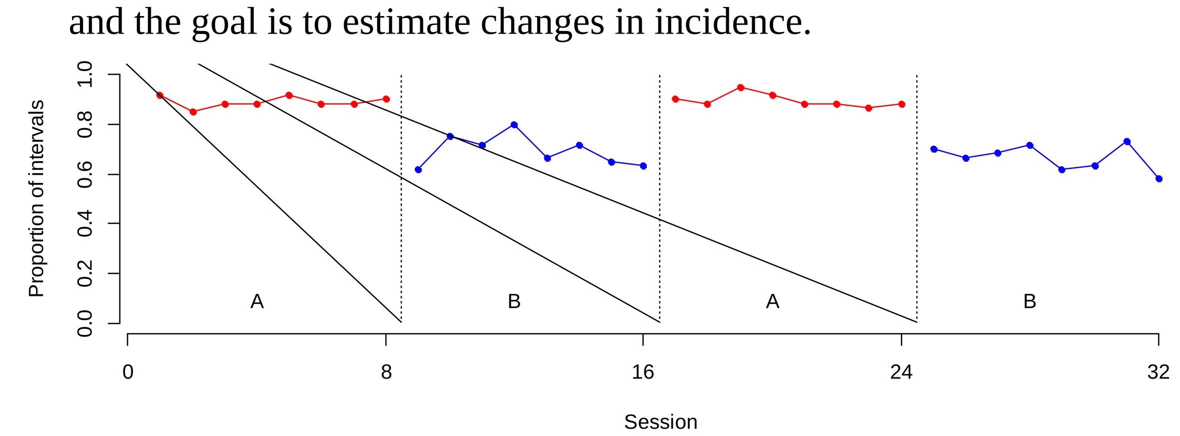
Variance estimate: $V_R = \dfrac{s_0^2}{n_0 \tilde{y}_0} + \dfrac{s_1^2}{n_1 \tilde{y}_1}$

$$\operatorname{logit}(x) = \ln(x) - \ln(1 - x)$$
$$\operatorname{cll}(x) = \ln(-\ln(1 - x))$$

Ignoring the problem

Consider a single-case study evaluating the effect of a teaching technique on the disruptive behavior of a student. The goal is to estimate/test change in prevalence.

- Study uses an ABAB design with 15-s PIR.
- Prior to intervention, $F_0(x) = \Gamma(x; 2,3)$, $\mu_0 = 6$, $G_0(x) = \Gamma(x; 3,4)$, $\lambda_0 = 12$, and prevalence is $\phi_0 = 1 / 3$.
- After intervention, $F_1(x) = \Gamma(x; 2,10)$, $\mu_1 = 20$, $G_1(x) = \Gamma(x; 3,10)$, $\lambda_1 = 30$, and prevalence is $\phi_1 = 2 / 5$.
- True prevalence has increased by 20% (i.e., the intervention is harmful).
- Yet 15-s PIR gives the impression that the intervention is effective in decreasing the prevalence of disruptive behavior.
- Similar examples can be found when the behavior has very short duration and the goal is to estimate changes in incidence.



Method 1: Long event durations

The upward bias of PIR as a measure of prevalence will be smaller if the average event duration is relatively long compared to the interval length. Formally, assume that $\mu_0, \mu_1 \geq \mu_{\min}$ for known $\mu_{\min}$. Then

$$\frac{\mu_{min}}{\mu_{min} + c} E\left(Y_s\right) < \phi_s < E\left(Y_s\right)$$

It follows that an approximate 95% CI for the log-prevalence ratio is

$$R \pm \left[\ln\left(\mu_{min} + c\right) - \ln\left(\mu_{min}\right) + z_{\alpha/2}\sqrt{V_R}\right]$$

Example 1: Moes (1998) used an ABAB/BABA design to evaluate effects of providing choice-making opportunities during homework tutoring on the disruptive behavior of four autistic students. Behavior was measured with 10 s PIR for $n_0 = n_1 = 10$ sessions in each condition. We assume $\mu_{\min} = 10$ s.

Case	y_0	s_0	y_1	s_1	95% CI for log-prevalence ratio
Carl	0.14	0.03	0.04	0.11	[-3.69, 1.24]
Charles	0.35	0.20	0.01	0.03	[-5.56, -1.30]
Chuck	0.27	0.15	0.09	0.13	[-2.80, 0.57]
James	0.50	0.17	0.03	0.09	[-5.39, -0.24]
FE Meta-analysis					[-3.27, -0.51]

Method 2: Short events, long interim times

PIR is sometimes also used in contexts where individual behavioral events are short. In this circumstance, if the interim times between events tend to be longer than the active interval length, then the number of intervals with behavior will closely approximate the total number of events. Formally, assume that that $\mu_0, \mu_1 \leq \mu_{\max}$ for known $\mu_{\max}$ and that $G_s(c; \lambda_s) \leq p$ for s = 0, 1 and known $p < 1$. Then

$$\frac{E\left(Y_s\right)}{\mu_{\max} + c} < \zeta_s < \frac{E\left(Y_s\right)}{(1-p)c}$$

It follows that an approximate 95% CI for the log-incidence ratio is

$$R \pm \left[\ln\left(\mu_{max} + c\right) - \ln\left(1 - p\right) - \ln(c) + z_{\alpha/2}\sqrt{V_R}\right]$$

Example 2: Dunlap et al. (1994) used an ABAB/ABA design to evaluate the effects of providing choice between academic activities on the disruptive behavior of three elementary school students with emotional and behavioral disorders. Behavior was measured with 10 s PIR for Sven and Ahmad and 15 s for the Wendell. We assume p = .15 for Sven and Ahmad and p = .25 for Wendell, and that $\mu_{\max}$ = 10s for all three.

Case	n_0	y_0	s_0	n_1	y_1	s_1	95% CI for log-incidence ratio
Ahmad	8	0.7	0.18	8	0.02	0.02	[-4.72, -1.96]
Sven	15	0.36	0.19	6	0.09	0.1	[-3.07, 0.39]
Wendall	10	0.26	0.12	11	0.06	0.07	[-3.01, 0.12]
FE Meta-analysis							[-3.69, -1.23]

Case	Prevalence estimates				Incidence estimates (per interval)			
	No Choice	Choice	log-ratio	95% CI	No Choice	Choice	log-ratio	95% CI
Moes (1998)								
Carl	0.001	0.039	3.429	[-6.137, 4.446]	0.146	0.001	-4.610	[-5.154, -2.828]
Charles	0.329	0.009	-3.631	[-8.308, -2.403]	0.019	0.003	-2.019	[-3.446, -0.617]
Chuck	0.247	0.084	-1.085	[-2.665, -0.203]	0.024	0.005	-1.541	[-3.050, -0.222]
James	0.465	0.029	-2.787	[-8.346, -1.559]	0.035	0.001	-3.244	[-4.422, -1.599]
Dunlap et al. (1994)								
Ahmad	0.676	0.002	-5.783	[-9.750, -3.500]	0.029	0.025	-0.143	[-2.177, 0.897]
Sven	0.333	0.081	-1.420	[-3.197, -0.617]	0.029	0.014	-0.716	[-2.091, 0.815]
Wendell	0.188	0.045	-1.428	[-2.885, -0.200]	0.071	0.016	-1.522	[-2.900, -0.387]

Method 3: Constant mean event duration, exponential interim time distribution

The first two methods make assumptions only about the mean event duration and the probability of short interim times, but not about the full distribution of event durations or interim times. Entertaining stronger distributional assumptions about the behavior stream will yield narrower bounds for parameters of interest. Specifically, assume that

- mean event durations in each sample are equal, $\mu_0 = \mu_1$ and
- interim times in each sample follow exponential distributions.

It follows that

$$f_L\left[E(Y_0), E(Y_1)\right] < \ln\left(\frac{\lambda_0}{\lambda_1}\right) < f_U\left[E(Y_0), E(Y_1)\right]$$

where

$$f_L\left(x, y\right) = \begin{cases} \operatorname{logit}(y) - \operatorname{logit}(x) & \text{if} \quad x > y \\ \operatorname{cll}(y) - \operatorname{cll}(x) & \text{if} \quad x \leq y \end{cases}$$

$$f_U\left(x, y\right) = \begin{cases} \operatorname{cll}(y) - \operatorname{cll}(x) & \text{if} \quad x > y \\ \operatorname{logit}(y) - \operatorname{logit}(x) & \text{if} \quad x \leq y \end{cases}$$

These bounds can be estimated by substituting sample means for the expectations. See the conference paper for details regarding variance estimation and confidence interval construction.

Example 1 continued: Returning to the study by Moes (1998), we assume the investigators are confident the that the choice-making intervention did not alter the average length of the participant's disruptive behavior. We further assume that the interim times between behaviors are exponentially distributed.

Case	Estimate bounds for log-interval ratio	95% CI for log-interval ratio
Carl	(-1.33, -1.28)	[-3.18, 0.57]
Charles	(-3.85, -3.63)	[-5.36, -2.16]
Chuck	(-1.34, -1.23)	[-2.47, -0.17]
James	(-3.47, -3.12)	[-5.44, -1.20]
FE Meta-analysis	(-2.26, -2.08)	[-3.01, -1.36]

Method 4: Moment estimators based on an Alternating Poisson Process

The first three methods all involve bounds (rather than point estimates) for the target parameters. Yet stronger parametric assumptions are required to obtain point estimates. Assume that the behavior stream follows an Alternating Poisson Process, in which event durations and interim times both follow exponential distributions (with separate means). It follows that

$$E\left(Y_s\right) = 1 - (1 - \phi_s)\exp\left(\frac{-\zeta_s c}{(1 - \phi_s)}\right)$$

$$Var\left(Y_s\right) = \frac{E(Y_s)\left[1 - E(Y_s)\right]}{K}\left[1 + \frac{2\phi_s}{KE(Y_s)}\sum_{k=1}^{K-1}(K - k)\exp\left(\frac{\zeta_s c}{\phi_s} - \frac{\zeta_s kL}{\phi_s(1 - \phi_s)K}\right)\right]$$

Moment estimators for prevalence and incidence can be obtained for each s = 0,1 by replacing the expectation and variance with corresponding sample moments, then solving for ϕ_s and ζ_s. Variance estimates and confidence intervals can be obtained via parametric bootstrapping.

Examples 1& 2 continued: Returning to the studies by Moes (1998) and Dunlap et al. (1994), we assume the behavior streams follow an Alternating Poisson Process.