Empirical Benchmarks for Between-Case Standardized Mean Differences from Single-Case Multiple Baseline Designs Examining Academic Interventions

James E. Pustejovsky
Man Chen
David Klingbeil
Ethan van Norman

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Man Chen

Ethan van Norman

David Klingbeil

Outline

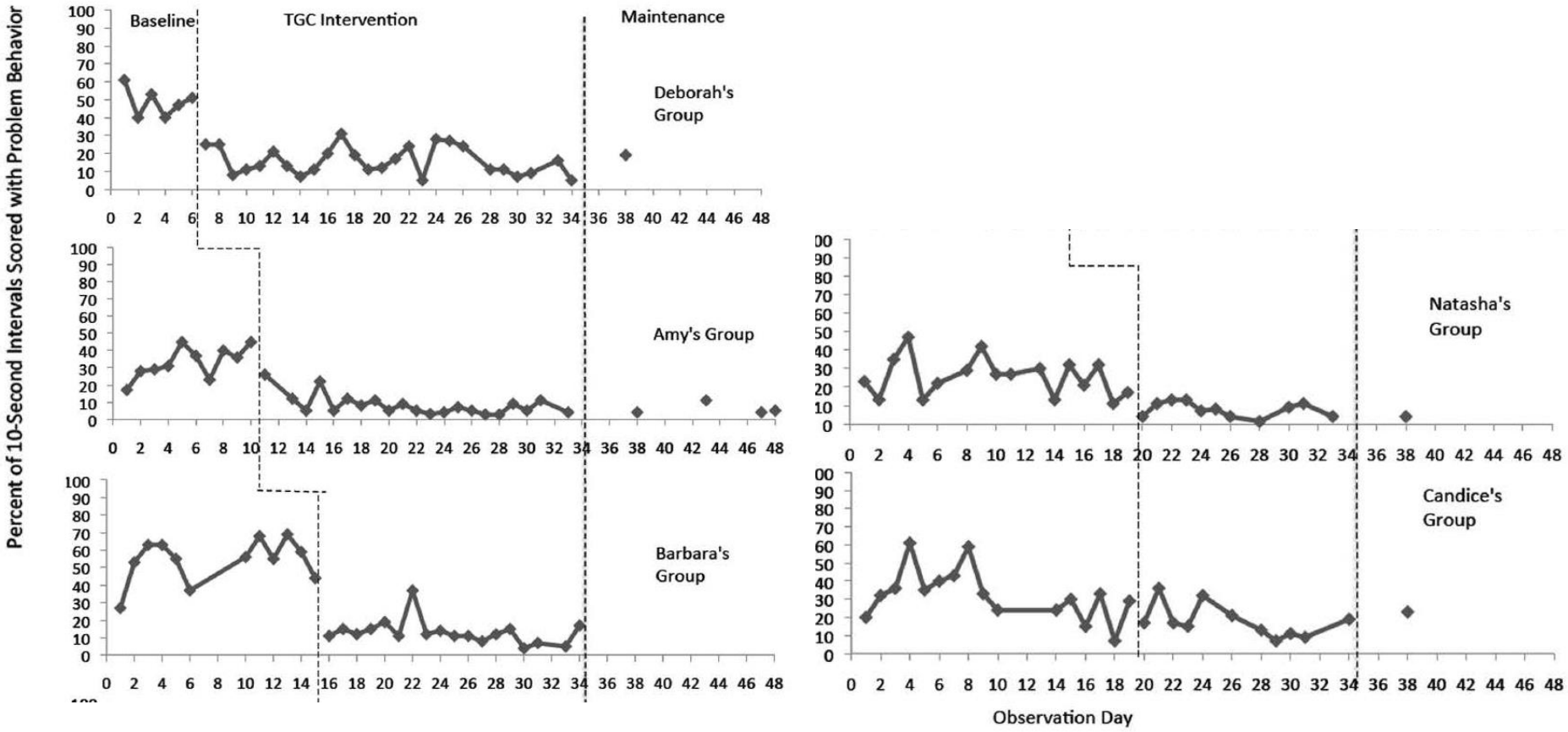
- **Single-case multiple baseline designs**
- Between-case standardized mean differences
- Systematic review of academic multiple baseline designs
- Analytic strategy
- Empirical benchmarks

Single-case multiple baseline designs

- Used for investigating effects of interventions / practices for individual participants across a variety of settings

- Essential features of multiple baseline designs
  - One or small number of participants
  - Repeated measurement of outcomes on each individual participant
  - Researcher controls introduction of intervention for each participant
  - Intervention initiation is staggered in time

Rodriguez & Anderson (2014). Integrating a social behavior intervention during small group academic instruction using a total group criterion intervention



Design schematic

A multiple baseline design across four participants

T1	T2	T3	T4	T5	T6	T7	T8	T9	T10	T11	T12	T13	T14	T15	T16	T17	T18
X	X	X	X	X	T	X	X	X	X	X	X	X	X	X	X	X	X
X	X	X	X	X	X	X	X	T	X	X	X	X	X	X	X	X	X
X	X	X	X	X	X	X	X	X	X	X	T	X	X	X	X	X	X
X	X	X	X	X	X	X	X	X	X	X	X	X	X	T	X	X	X	X

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Between-case standardized mean difference (BC-SMD)

- Shadish, Rindskopf, & Hedges (2008) asked:

  Can we estimate an effect size based on the data from a single-case design that is in the same metric as the standardized mean difference effect size from a between-groups design?

- Why do this? (Shadish, Hedges, Horner, & Odom, 2015)
  - **Translation** of single-case research for researchers who work primarily with between-groups designs.
  - **Comparison** of results from single-case studies and between-groups studies, for purposes of understanding the utility and limitations of each type of design.
  - **Synthesis** involving both single-case and between-groups designs.

SMD in between-group experiments

- What is the SMD from a between-groups experiment?

$$\delta_{BC} = \frac{\begin{pmatrix}\text{Average outcome if}\\ \text{everybody gets treatment}\end{pmatrix} - \begin{pmatrix}\text{Average outcome if}\\ \text{nobody gets treatment}\end{pmatrix}}{\begin{pmatrix}\text{Outcome standard dev.}\\ \text{nobody gets treatment}\end{pmatrix}}$$

$$\delta_{BC} = \frac{\begin{pmatrix}\text{Average outcome if}\\ \text{everybody gets treatment}\end{pmatrix} - \begin{pmatrix}\text{Average outcome if}\\ \text{nobody gets treatment}\end{pmatrix}}{\sqrt{\begin{pmatrix}\text{Within participant}\\ \text{variance}\end{pmatrix} + \begin{pmatrix}\text{Between participant}\\ \text{variance}\end{pmatrix}}}$$

- These quantities can be estimated from multiple baseline design data using a hierarchical linear model.
  - We'll need to have a sample of multiple participants (bare minimum of 3, more for more complex models).
  - We'll need to be specific about ***timing*** of intervention and follow-up.

Estimating BC-SMDs: The broad strategy

Pustejovsky, Hedges, and Shadish (2014):

1. Develop a hierarchical linear model that describes
   a) the form of time trends and intervention effects
   b) how the trends and intervention effects ***vary across cases***.

2. Imagine a ***hypothetical between-groups experiment*** with the same population of participants, same treatment, same outcomes.
   - When is treatment initiated?
   - When are outcomes assessed?

3. Use the hierarchal model to estimate the between-case SMD for the hypothetical experiment.

Design translation

A multiple baseline design

T1	T2	T3	T4	T5	T6	T7	T8	T9	T10	T11	T12	T13	T14	T15	T16	T17	T18
X	X	X	X	X	T	X	X	X	X	X	X	X	X	X	X	X	X
X	X	X	X	X	X	X	X	T	X	X	X	X	X	X	X	X	X
X	X	X	X	X	X	X	X	X	X	X	T	X	X	X	X	X	X
X	X	X	X	X	X	X	X	X	X	X	X	X	X	T	X	X	X

A hypothetical between-group design (with pre-test)

T1	T2	T3	T4	T5	T6	T7	T8	T9	T10	T11	T12	T13	T14	T15	T16	T17	T18
				X	T										X		
				X	T										X		
				X											X		
				X											X		

Motivation

- BC-SMD has been used in many primary single-case design studies, as well as many systematic reviews of single-case research.
- What Works Clearinghouse recently adopted BC-SMDs for summarizing findings from single-case designs.
- But no reference benchmarks available.
  - Theoretically comparable to between-group effect sizes.
  - But multiple baselines are used in different contexts that group designs, so existing group design benchmarks are probably not be appropriate.

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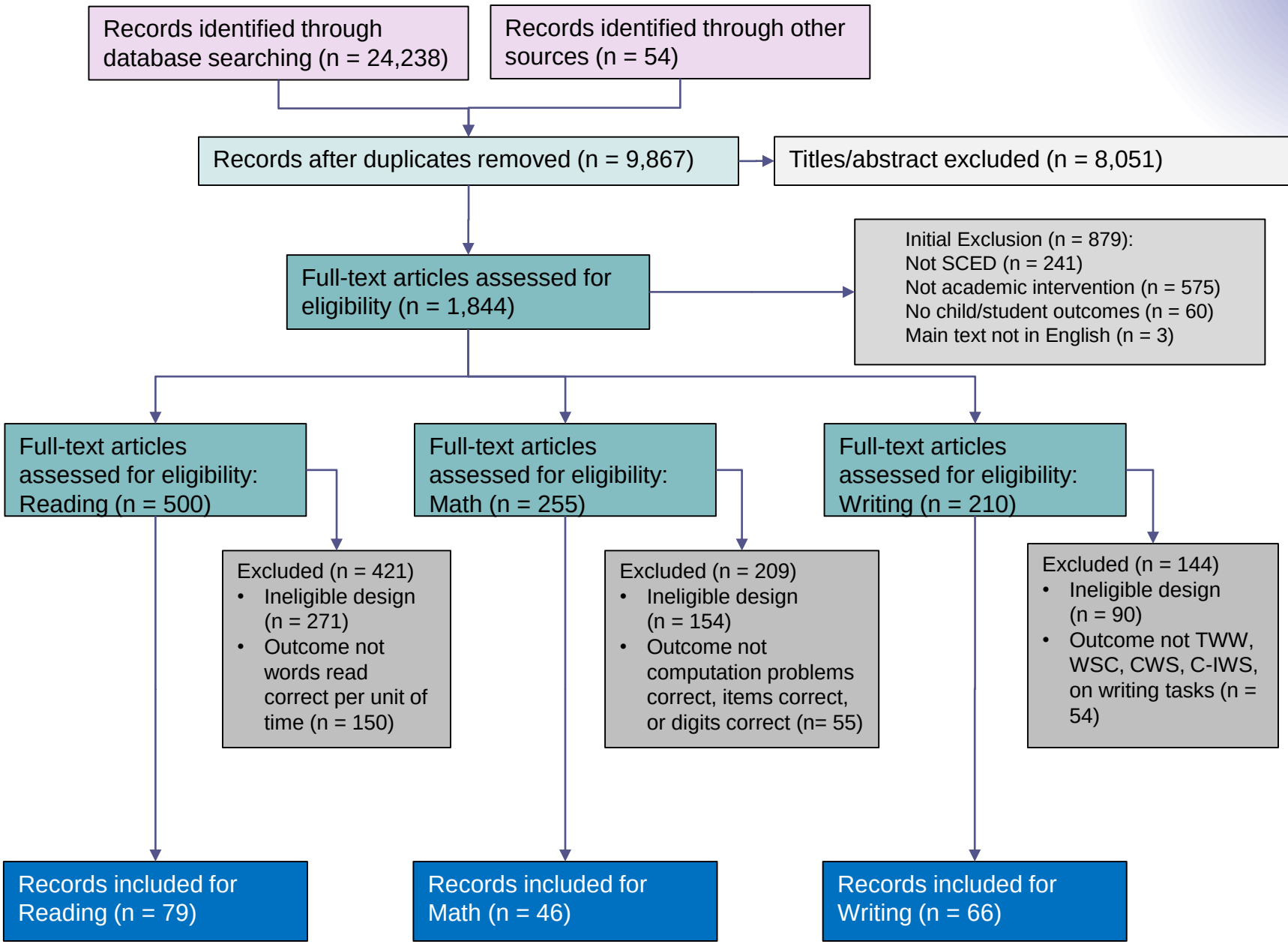
Inclusion criteria and search strategy

- **Design:** Across-participant multiple baseline design with 3+ participants.
- **Participants:** Students in pre-kindergarten through 12th grade (or Special Education up to age 21)
- **Intervention:** Any intervention targeting an academic skill
- **Comparison:** Baseline prior to intervention
- **Outcomes:** Specific, curriculum-based measures of math, reading, or writing

- Databases: Academic Search Complete, ERIC, PsycInfo

- Search string: “single-case” AND (“read*” OR “math*” OR “writ*” OR “spell*” OR “academic*” OR “learn*”)


Records identified through database searching (n = 24,238)
Records identified through other sources (n = 54)
Records after duplicates removed (n = 9,867)
Titles/abstract excluded (n = 8,051)
Full-text articles assessed for eligibility (n = 1,844)
Initial Exclusion (n = 879):
Not SCED (n = 241)
Not academic intervention (n = 575)
No child/student outcomes (n = 60)
Main text not in English (n = 3)
Full-text articles assessed for eligibility: Reading (n = 500)
Full-text articles assessed for eligibility: Math (n = 255)
Full-text articles assessed for eligibility: Writing (n = 210)
Excluded (n = 421)
• Ineligible design (n = 271)
• Outcome not words read correct per unit of time (n = 150)
Excluded (n = 209)
• Ineligible design (n = 154)
• Outcome not computation problems correct, items correct, or digits correct (n= 55)
Excluded (n = 144)
• Ineligible design (n = 90)
• Outcome not TWW, WSC, CWS, C-IWS, on writing tasks (n = 54)
Records included for Reading (n = 79)
Records included for Math (n = 46)
Records included for Writing (n = 66)

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Estimating BC-SMD effect sizes

- Initial visual analysis of every included design to determine appropriate functional forms for modeling.

- Effect size estimates generated using scdhlm R package (Pustejovsky, Chen, & Hamilton, 2022) based on a model with
  - Linear time trends for baseline phases
  - Intervention-by-time interactions
  - Random intercepts (but no random slopes)
  - Auto-correlated errors (AR1)

- Hypothetical between-group design parameters (defaults)
  - Intervention time equal to actual intervention time for first participant
  - Follow-up time based on the length of the shortest intervention phase

Summarizing distribution of effect sizes

- Empirical distribution of effect size estimates
- Meta-analytic model
  - Multi-level random effects
  - Prediction intervals for center of distribution
- Distribution of Empirical Bayes estimates
  - Non-parametric bootstrap intervals

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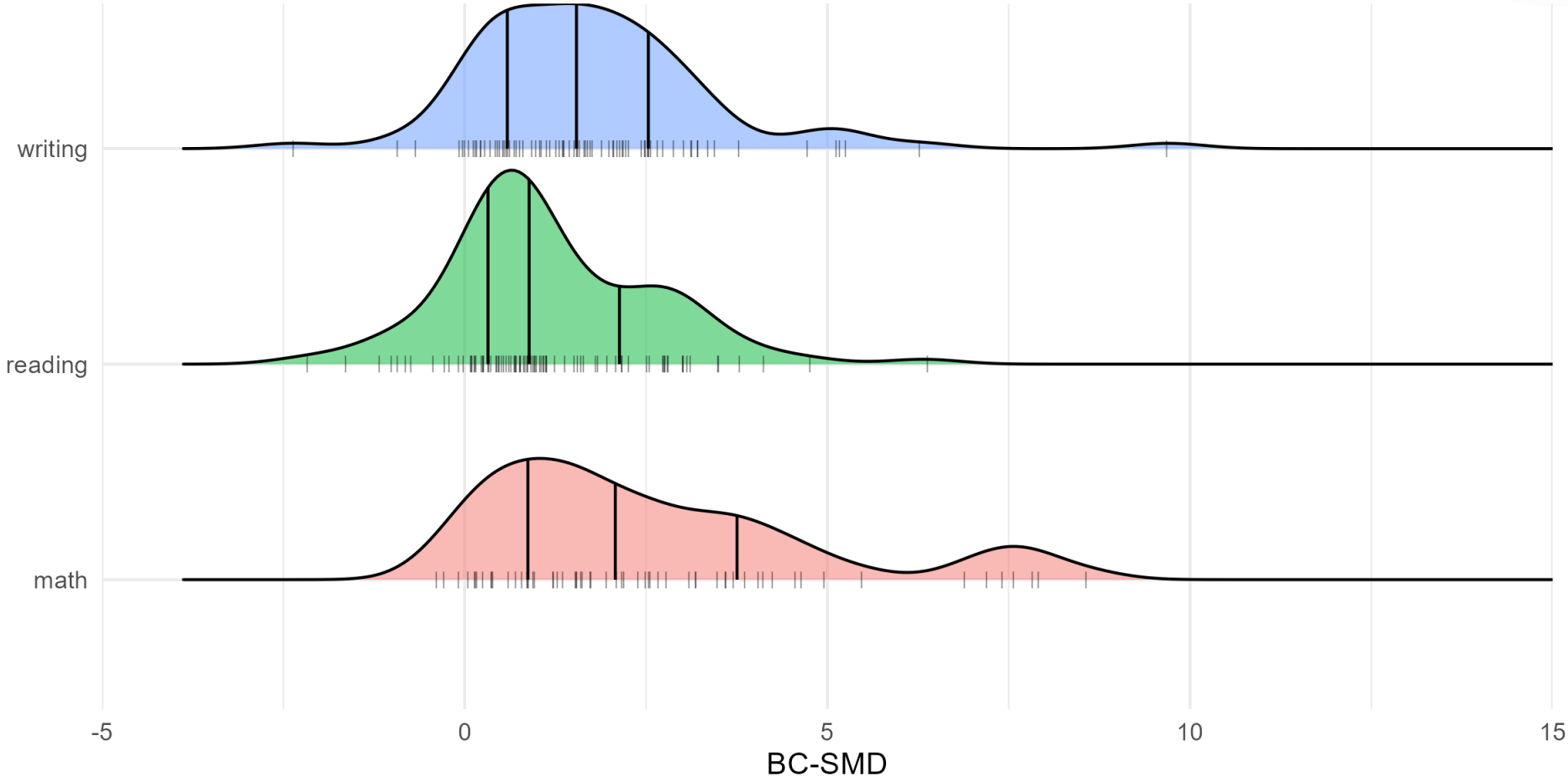
Empirical densities



Meta-analysis and empirical Bayes

Domain	Estimator	10th	20th	30th	40th	50th	60th	70th	80th	90th
Math	Percentile	0.21	0.77	1.10	1.54	2.08	2.59	3.49	4.20	7.13
	Meta-Analysis	-0.21	0.71	1.37	1.93	2.45	2.97	3.53	4.19	5.11
	Empirical Bayes	0.24	0.74	1.21	1.67	2.12	2.56	3.37	4.22	6.67
Reading	Percentile	-0.25	0.19	0.44	0.48	0.89	1.10	1.65	2.51	3.01
	Meta-Analysis	-0.05	0.25	0.46	0.64	0.81	0.98	1.16	1.37	1.67
	Empirical Bayes	0.11	0.39	0.57	0.73	0.84	0.94	1.09	1.29	1.60
Writing	Percentile	0.14	0.49	0.77	1.30	1.54	1.97	2.22	2.67	3.30
	Meta-Analysis	0.10	0.59	0.95	1.25	1.53	1.81	2.11	2.47	2.96
	Empirical Bayes	0.30	0.74	0.96	1.33	1.55	1.81	2.05	2.40	3.02

Reference benchmarks

- Using middle 40% of distribution (30th-70th percentile)
- empirical Bayes estimates

Domain		30th		70th	
Math	Small	1.2	Medium	3.4	Large
Reading	Small	0.6	Medium	1.1	Large
Writing	Small	1.0	Medium	2.1	Large

Observations and limitations

- Compared to group designs, distributions of BC-SMD effects from single-case multiple baseline designs cover ***substantially larger values*** and are ***more dispersed***.

- Differences between designs could be due to differences in
  - Populations
  - Interventions
  - Dependent variables
  - Settings
  - Temporal horizons

- BC-SMDs from multiple baseline designs are sensitive to follow-up time.

- Critical to interpret findings within the context of the topic area and based on the logic of single-case designs.

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