Small-Sample Methods for Cluster-Robust Variance Estimation and Hypothesis Testing in Fixed Effects Models

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In brief…

- Standard cluster-robust variance estimators behave poorly when the number of clusters is small
- McCaffrey, Bell, & Botts (2001; Bell & McCaffrey, 2002) proposed "bias-reduced linearization" variance estimator (BRL)
  - Improves bias of standard errors when number of clusters is small
  - Satterthwaite degrees of freedom correction for t-tests
  - Breaks in models with fixed effects in multiple dimensions
- Our work:
  - Extends BRL so that it works in models with fixed effects
  - Develops an F-test for multi-parameter hypothesis tests
  - Provides easy-to-use software implementation in R

Fixed effects models

- Consider state-by-year panel data model

$$y_{it} = \mathbf{x}_{it}\boldsymbol{\delta} + \gamma_i + \zeta_t + e_{it} \qquad i = 1, \ldots, m; \quad t = 1, \ldots, T$$

$$\mathbf{Y}_i = \mathbf{X}_i\boldsymbol{\beta} + \mathbf{e}_i$$

- Common to treat γ_i, ζ_t as fixed effects, estimate $\boldsymbol{\beta}$ by OLS/FGLS
- Cluster standard errors to allow for further correlation among errors within each state.
  - Asymptotic Wald tests/t-tests with $m - 1$ degrees of freedom
  - These tests have excessive type-I error when m is small (Cameron & Miller, 2015; Imbens & Kolesar, 2015)
  - And there's no bright-line rule for "large enough"

Bias-reduced linearization

- McCaffrey, Bell, & Botts (2001) proposed a correction to $\mathbf{V}^{CR}$ based on a ***working model*** for the error covariance structure.

- The corrected variance estimator is a "fancy" sandwich:

$$\mathbf{V}^{BRL} = \left(\mathbf{X}^t\mathbf{X}\right)^{-1}\left(\sum_{j=1}^{m}\mathbf{X}_j^t\mathbf{A}_j\hat{\mathbf{e}}_j\hat{\mathbf{e}}_j^t\mathbf{A}_j^t\mathbf{X}_j\right)\left(\mathbf{X}^t\mathbf{X}\right)^{-1}$$

with adjustment matrices $\mathbf{A}_1,\ldots,\mathbf{A}_G$ chosen to satisfy

$$\mathrm{E}\left(\mathbf{V}^{BRL}\right) = \mathrm{Var}\left(\hat{\boldsymbol{\beta}}\right)$$

Fixed effects models

- BRL breaks when the model includes fixed effects in multiple dimensions (Angrist & Pischke, 2009; Young, 2016).
  - Requires inversion of rank-deficient matrices

- We demonstrate that the ***Moore-Penrose generalized inverse*** can be used to construct a variance estimator that is still unbiased under the working model.
  - Adjustment matrices can be calculated from least-squares dummy variable fit or from “within” estimation, after absorbing fixed effects

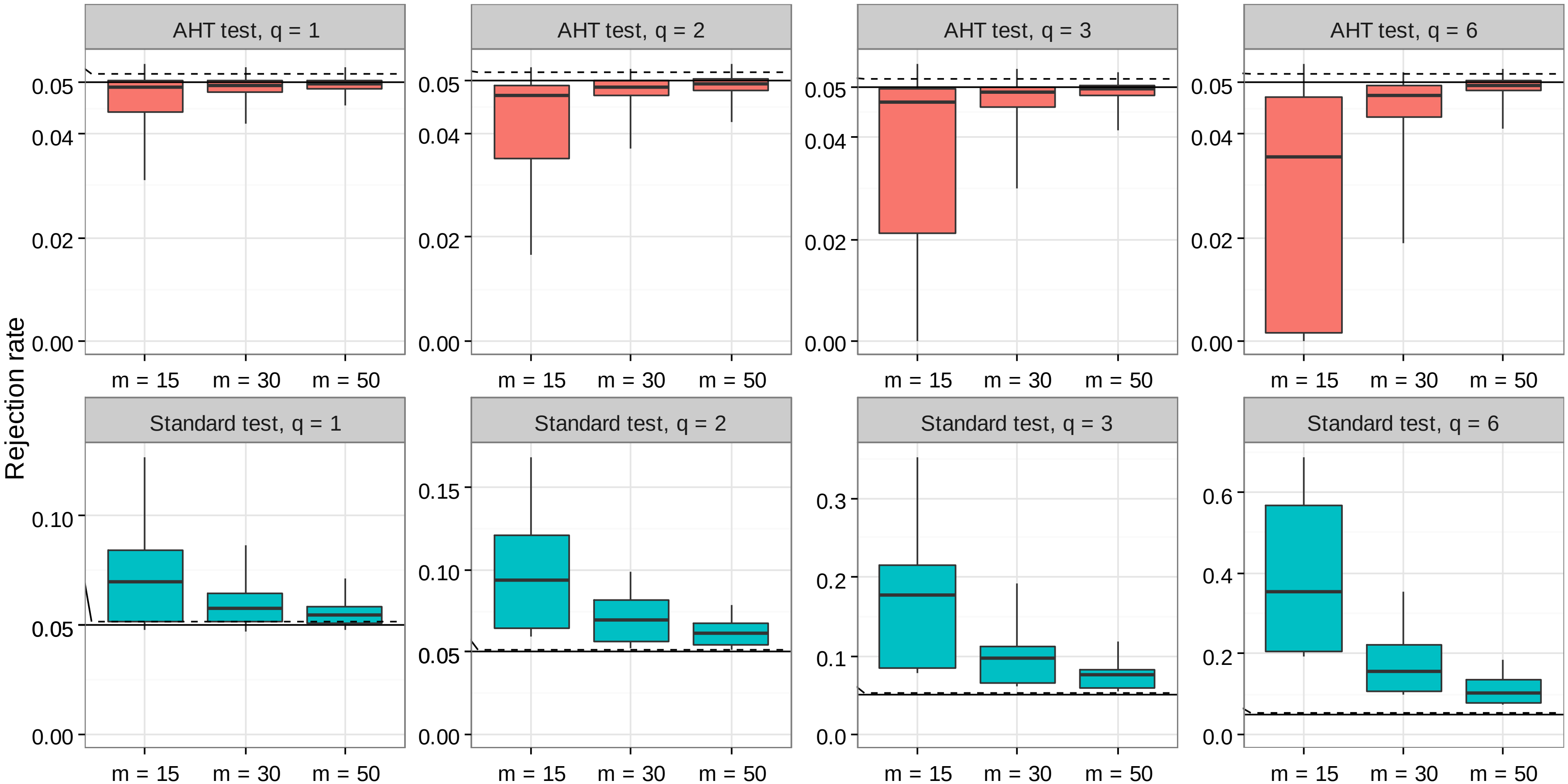
Approximate Hotelling Test

- We propose a generalization of the Satterthwaite approximation to the multi-dimensional case, with $H_0 : \mathbf{C}\boldsymbol{\beta} = \mathbf{0}$
- Approximate the distribution of V^{BRL} using a Wishart distribution with degrees of freedom *η*.
- Estimate *η* by matching mean and **total variance** of V^{BRL}.

$$F_{AHT} = \frac{\eta - q + 1}{\eta q} \left(\mathbf{C}\hat{\boldsymbol{\beta}}\right)^t \left(\mathbf{C}\mathbf{V}^{BRL}\mathbf{C}\right)^{-1} \left(\mathbf{C}\hat{\boldsymbol{\beta}}\right)$$

$$F_{AHT} \dot{\sim} F\left(q, \eta - q + 1\right)$$

AHT maintains close-to-nominal α



Angrist & Lavy (2009)

- Cluster-randomized trial in 40 high schools in Israel.
- Tested effects of monetary incentives on post-secondary matriculation exam (Bagrut) completion rates.
- Longitudinal data, difference-in-differences specification.
- Focus on effects for higher-achieving girls

Hypothesis	Test	F	df	p-value
treatment effect (q = 1)	Standard	5.746	34.00	.022
	Satterthwaite	5.169	18.13	.035
Moderation by school sector (q = 2)	Standard	3.186	34.00	.054
	AHT	1.665	7.84	.250

Software

- R package `clubSandwich`
  - Available on Comprehensive R Archive Network (v0.2.1)
  - Development version at https://github.com/jepusto/clubSandwich
- Works with a wide variety of fitted models
  - `lm` models: Ordinary/weighted least squares
  - `plm` package: Fixed-effects/random-effects panel models
  - `nlme` package: GLS and HLM models
  - Meta-analysis (`metafor` and `robumeta` packages)

Thank you

- pusto@austin.utexas.edu
- http://jepusto.github.io/
- Working paper available at http://arxiv.org/abs/1601.01981

References

- Angrist, J. D., & Lavy, V. (2009). The effects of high stakes high school achievement awards : Evidence from a randomized trial. *American Economic Review*, *99*(4), 1384–1414. doi:10.1257/aer.99.4.1384
- Angrist, J. D., & Pischke, J. (2009). *Mostly harmless econometrics: An empiricist's companion*. Princeton, NJ: Princeton University Press.
- Arellano, M. (1987). Computing robust standard errors for within-groups estimators. *Oxford Bulletin of Economics and Statistics*, *49*(4), 431–434.
- Bell, R. M., & McCaffrey, D. F. (2002). Bias reduction in standard errors for linear regression with multi-stage samples. *Survey Methodology*, *28*(2), 169–181.
- Cameron, A. C., & Miller, D. L. (2015). *A practitioner's guide to cluster-robust inference*.
- Carpenter, C., & Dobkin, C. (2011). The minimum legal drinking age and public health. *Journal of Economic Perspectives*, *25*(2), 133–156. doi:10.1257/jep.25.2.133
- Cortes, K. E., Goodman, J. S., & Nomi, T. (2015). Intensive math instruction and educational attainment: Long-run impacts of double-dose algebra. *Journal of Human Resources, 50*(1), 108–158. doi:10.3386/w20211
- Imbens, G. W., & Kolesar, M. (2015). *Robust standard errors in small samples: Some practical advice*. Retrieved from https://www.princeton.edu/~mkolesar/papers/small-robust.pdf
- Lee, D. S., & Card, D. (2008). Regression discontinuity inference with specification error. *Journal of Econometrics*, *142*(2), 655–674. doi:10.1016/j.jeconom.2007.05.003
- Liang, K.-Y., & Zeger, S. L. (1986). Longitudinal data analysis using generalized linear models. *Biometrika*, *73*(1), 13–22.
- McCaffrey, D. F., Bell, R. M., & Botts, C. H. (2001). Generalizations of biased reduced linearization. In *Proceedings of the Annual Meeting of the American Statistical Association*.
- Young, A. (2016). *Improved, nearly exact, statistical inference with robust and clustered covariance matrices using effective degrees of freedom corrections*.