META-ANALYSIS OF DEPENDENT EFFECT SIZES: A REVIEW AND CONSOLIDATION OF METHODS

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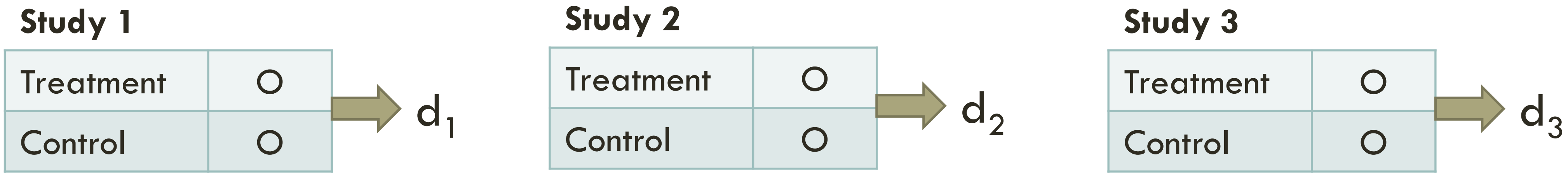
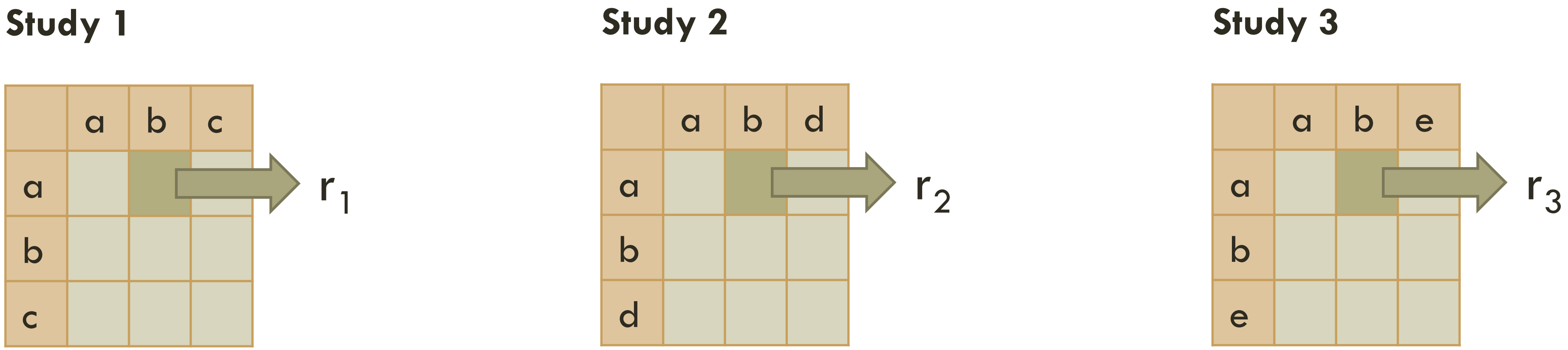
BASIC META-ANALYSIS METHODS ASSUME INDEPENDENT EFFECT SIZES

In a meta-analysis of experiments:



In a meta-analysis of correlations:



BUT DEPENDENT EFFECT SIZES ARE VERY COMMON IN PRACTICE

Multiple outcomes measured on common set of participants

Treatment	O P Q
Control	O P Q

d_{O1}
d_{P1}
d_{Q1}

Outcome measured at multiple follow-up times

Treatment	O_1	O_2	O_3
Control	O_1	O_2	O_3

d_{12}
d_{22}
d_{32}

Multiple treatment conditions compared to a common control

Treatment T	O
Treatment U	O
Control	O

d_{T3}
d_{U3}

Multiple correlations from a common sample

	a	b	c	d
a				
b				
c				
d				

r_{ab}
r_{ac}
r_{ad}

FRIESE, FRANKENBACH, JOB, & LOSCHELDER (2017). DOES SELF-CONTROL TRAINING IMPROVE SELF-CONTROL: A META-ANALYSIS.

33 experimental studies, 166 effect size estimates (standardized mean differences)

- ✓ Multiple outcomes (1-13 outcomes per study, median = 2)
- ✓ Multiple follow-up times (immediate post-test and/or later follow-up)
- ✓ Multiple treatment conditions (1-4 treatment conditions per study)
- ✓ Multiple control conditions (active and/or passive control)
- ✓ 1-52 effect size estimates per study (median = 2)

CORRELATIONS BETWEEN ES ESTIMATES

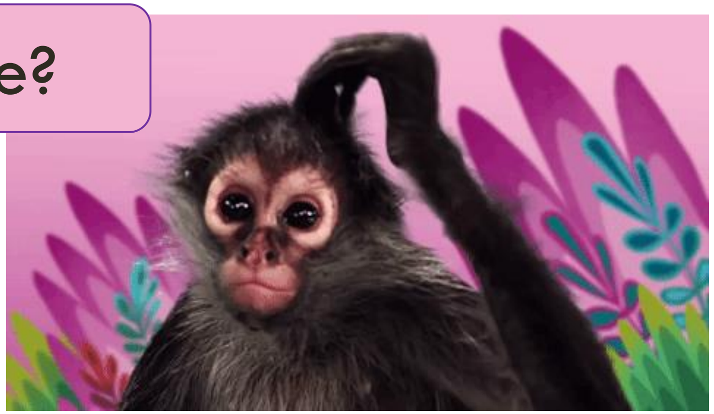
Multiple treatments compared to common control

- known formulas (Gleser & Olkin, 2009), easy enough to calculate

- Multiple outcomes/multiple follow-ups
  - known formulas (Gleser & Olkin, 2009)
  - require knowing correlations among outcomes/repeated measures (often not available)

- Multiple correlations from common sample
  - known, icky formulas (Steiger, 1980)
  - need to know correlations between ALL variables involved

Which one should I use?



METHODS FOR HANDLING DEPENDENCE

Becker (2000) described four broad strategies:

- **Ignore**
- **Combine**
  - aggregated effects (Borenstein et al., 2009)
  - Shifting unit-of-analysis (Cooper, 1998)
- **Sub-classify**
  - Shifting unit-of-analysis (Cooper, 1998)
- **Model**
  - Robust variance estimation (Hedges, Tipton, & Johnson, 2010)
  - Multivariate meta-analysis (Raudenbush, Becker, & Kalaian, 1988; Kalaian & Raudenbush, 1996)
  - Multi-level meta-analysis (Van den Noortgate et al., 2013, 2015)

RE-ANALYSIS OF SELF-CONTROL TRAINING STUDIES

	(1) Aggregated effects	(2) Shifting unit-of-analysis	(3) Multivariate meta-analysis	(4) Multi-level meta-analysis	(5) Robust variance estimation
Overall Average ES (k = 33, N = 166)	0.281*** [0.059]		0.261*** [0.052]	0.263*** [0.054]	0.289*** [0.060]
Between-study SD	0.207		0.202	0.254	0.289
Within-study SD			0.143	0.027	
Moderator analysis by type of outcome					
Stamina (k = 16, N = 31)	0.579*** [0.157]	0.413*** [0.093]	0.359*** [0.077]	0.351*** [0.071]	0.579*** [0.123]
Strength (k = 28, N = 135)	0.199** [0.071]	0.171** [0.064]	0.236*** [0.054]	0.238*** [0.055]	0.203** [0.065]
Difference	-0.380* [0.185]	-0.243* [0.113]	-0.123 [0.072]	-0.112 [0.059]	-0.376* [0.136]

AGGREGATED EFFECTS

- Average estimates to generate single “synthetic” ES per study.

- Estimating variance of synthetic ES requires correlations among component ES (Borenstein et al., 2009).
  - Common to use a rough approximation assuming $r \approx 1$.

- Limits moderator/meta-regression analyses to between-study predictors.

SUB-GROUPS/SHIFTING UNIT-OF-ANALYSIS

- If ES can be classified into sub-groups where each study contributes ≤ 1 ES estimate, then univariate meta-analysis can be conducted within sub-groups.

- If there are still multiple ES per sub-group, aggregate (Cooper, 1998).
  - Need correlations between effects within sub-group in order to get variances of aggregated effects.
- Average effects by sub-group are not independent.
  - How to make comparisons between average effects by sub-group?
- Different ES estimates for each moderator analysis.
  - How to do meta-regression with multiple predictors?

MULTIVARIATE META-ANALYSIS

(Raudenbush, Becker, & Kalaian,1988; Kalaian & Raudenbush, 1996)

- Hierarchical model for component ES estimates nested within studies

$$T_{ij} = \mathbf{x}_{ij}\boldsymbol{\beta} + u_j + v_{ij} + e_{ij}$$

where $u_j \sim N(0, \tau^2)$, $v_{ij} \sim N(0, \omega^2)$, $e_{ij} \sim N(0, s_{ij}^2)$, $\boxed{\mathrm{Cov}\left(e_{hj}, e_{ij}\right) = r_{hij} s_{hj} s_{ij}.}$

- Requires estimates/assumptions about ES correlations r_{hij}.
  - In the example, I calculated *r* for multiple T-common C studies, assumed r = 0.17 for multiple outcomes/time-points.
- Allows for modeling of between- and within-study variation in the ES.
- Makes use of between- and within-study variation in predictors.

MULTI-LEVEL META-ANALYSIS

(Van den Noortgate, López-López, Marín-Martínez, & Sánchez-Meca, 2013, 2015)

- Use multi-level model to account for dependence between ES estimates within studies, ***ignoring the sampling correlations***:

$$T_{ij} = \mathbf{x}_{ij}\boldsymbol{\beta} + u_j + v_{ij} + e_{ij}$$

where $u_j \sim N(0, \tau^2)$, $v_{ij} \sim N(0, \omega^2)$, $e_{ij} \sim N(0, s_{ij}^2)$, $\boxed{\mathrm{Cov}(e_{hj}, e_{ij}) = 0.}$

- Simulation evidence indicates that this approach can be "robust" to mis-specified correlation structure.
- But unclear whether robustness holds generally.

ROBUST VARIANCE ESTIMATION

(Hedges, Tipton, & Johnson, 2010)

- Meta-analysis/meta-regression using “sandwich” variance estimation methods
  - robust to mis-specified/unknown correlations between ES within studies.
  - sandwich estimation methods apply to very general class of models.
- RVE implementation involves
  - choosing between “correlated effects” or “hierarchical effects” working models.
  - making “working” assumption about correlation between ES estimates.
- Uses semi-efficient diagonal weights:

$$w_{ij} = \frac{1}{n_j\left(\bar{s}_j^2 + \hat{\tau}^2\right)}, \qquad \text{where } \bar{s}_j^2 = \frac{1}{n_j}\sum_{i=1}^{n_j} s_{ij}^2$$

- Studies with more effects will get less weight in meta-regressions that have within-study predictors.

RE-ANALYSIS OF SELF-CONTROL TRAINING STUDIES

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COMPARISON

	Aggregated effects	Shifting unit-of-analysis	Multivariate meta-analysis	Multi-level meta-analysis	Robust variance estimation
Requires making "working" assumption about correlations	✓	✓	✓	✓	✓
Robustness to correlation assumptions	?	?	?	?	Robust
Meta-regression specification	Limited	Limited	Flexible	Flexible	Flexible
Random effects specification	Limited	Somewhat limited	Flexible	Flexible	Limited

CONSOLIDATION

- Robust “sandwich” variance estimation can be used with ***any*** of the methods.
- Default RVE weights should not be used for meta-regression with predictors that vary within study.
- Multi-level meta-analysis = multi-variate meta-analysis assuming r = 0.
- More attention to within- versus between-study variation in moderators.
- Improve computational tools to make multivariate meta-analysis easier to implement.

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