Some implications of behavioral observation procedures for meta-analysis of single-case research

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November 14, 2012

Defining & estimating effect sizes for single-case studies of free-operant behavior

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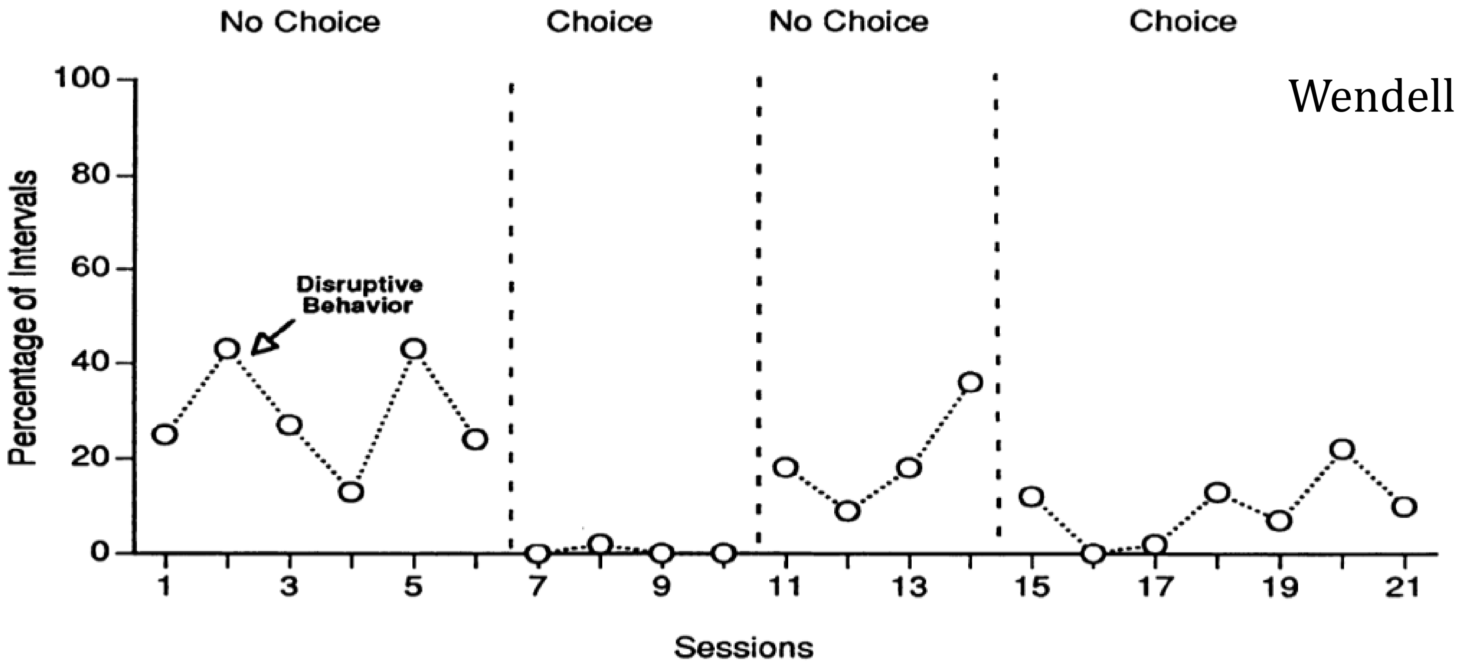
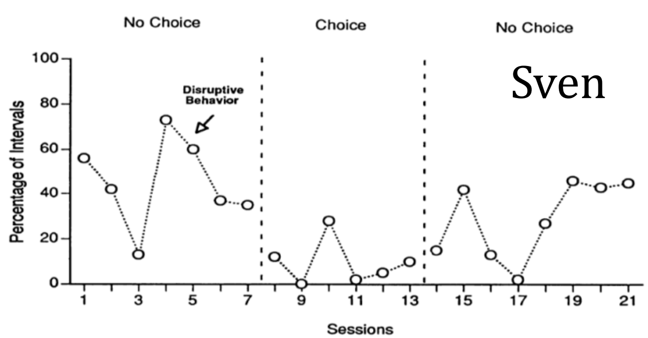
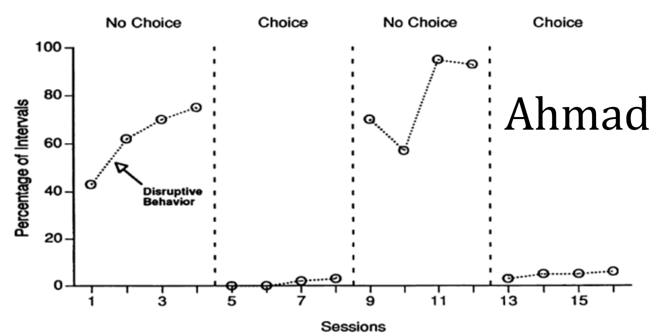
Dunlap, et al. (1994)

Choice making to promote adaptive behavior for students with emotional and behavioral challenges.







Single-case designs (SCDs)

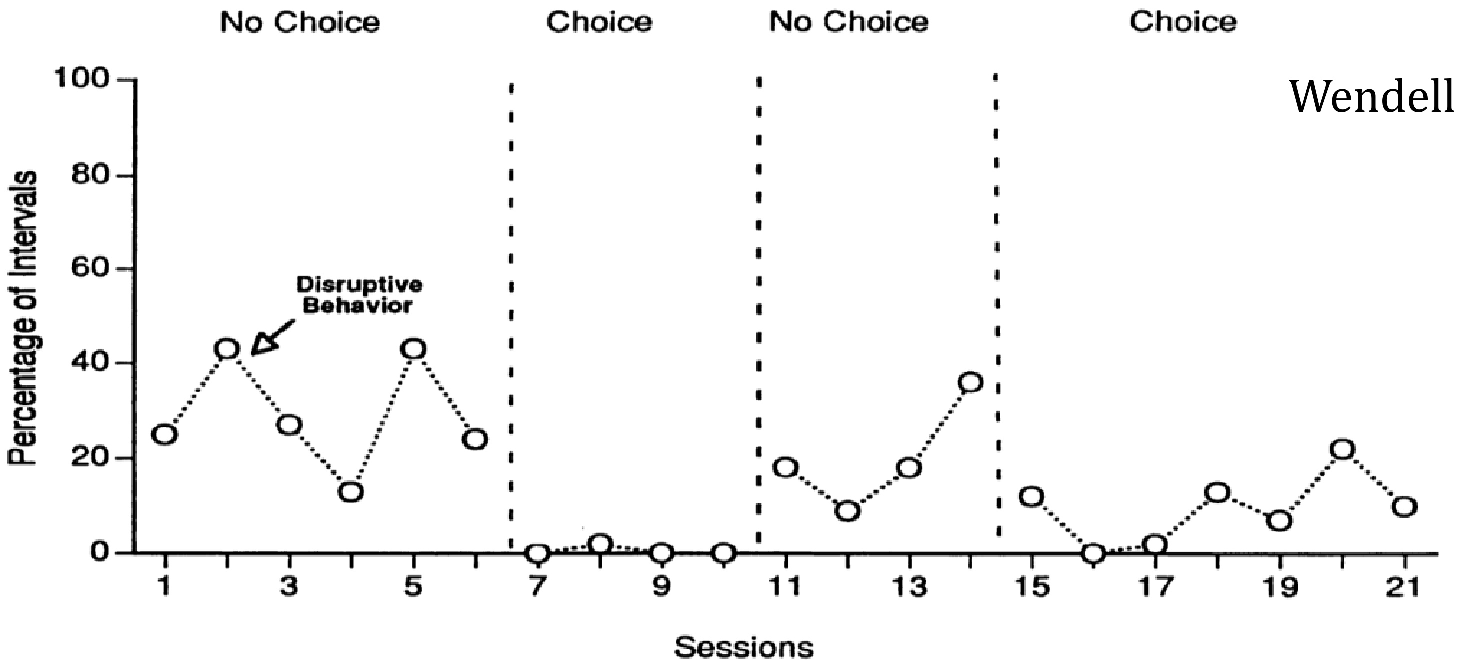
- Class of research methods for evaluating effects on individuals of practices, interventions, programs
- Applications in many areas of education research and psychology
- Repeated measurements of outcome variable(s)
- Deliberate, researcher-controlled manipulation of treatment

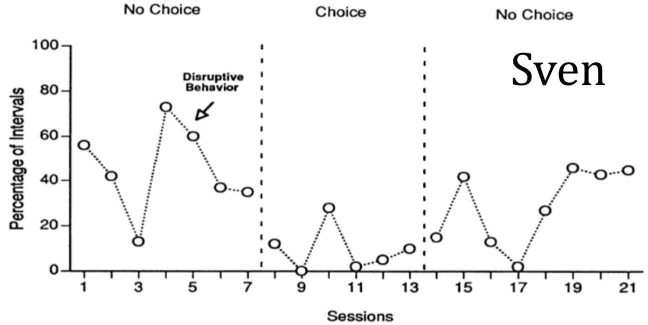
Outline

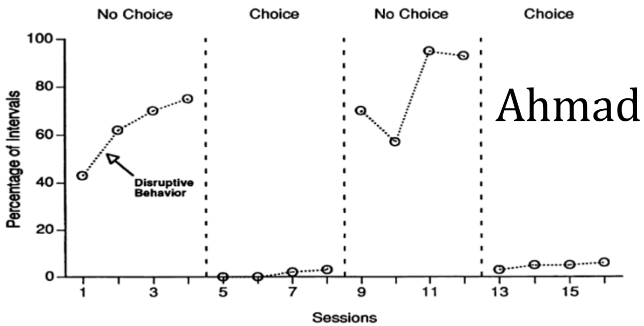
- **Background**
  on single-case research & effect sizes
- **A different approach to defining effect sizes**
  using a model for free-operant behavior
- **Estimation procedures**
  for partial interval data
- **Discussion**
  and future directions

Dunlap, et al. (1994)

Choice making to promote adaptive behavior for students with emotional and behavioral challenges.







Meta-analysis of single-case research

- Approach for summarizing many single-case studies
- Means for identifying evidence-based practices
- No consensus about appropriate methods

Effect sizes for single case designs

- Effect sizes
  - Basic units of analysis in a meta-analysis
  - Quantitative measures of study results (i.e., treatment effects)
  - What is the right metric for comparing results of studies that use different outcome measures?

- Many proposed effect size metrics for single-case designs (Beretvas & Chung, 2008)
  - Generic
  - Computational formulas, without reference to models

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Shogren, et al. (2004)

The effect of choice-making as an intervention for problem behavior

- Meta-analysis containing 13 studies (including Dunlap, et al., 1994)
- 32 unique cases

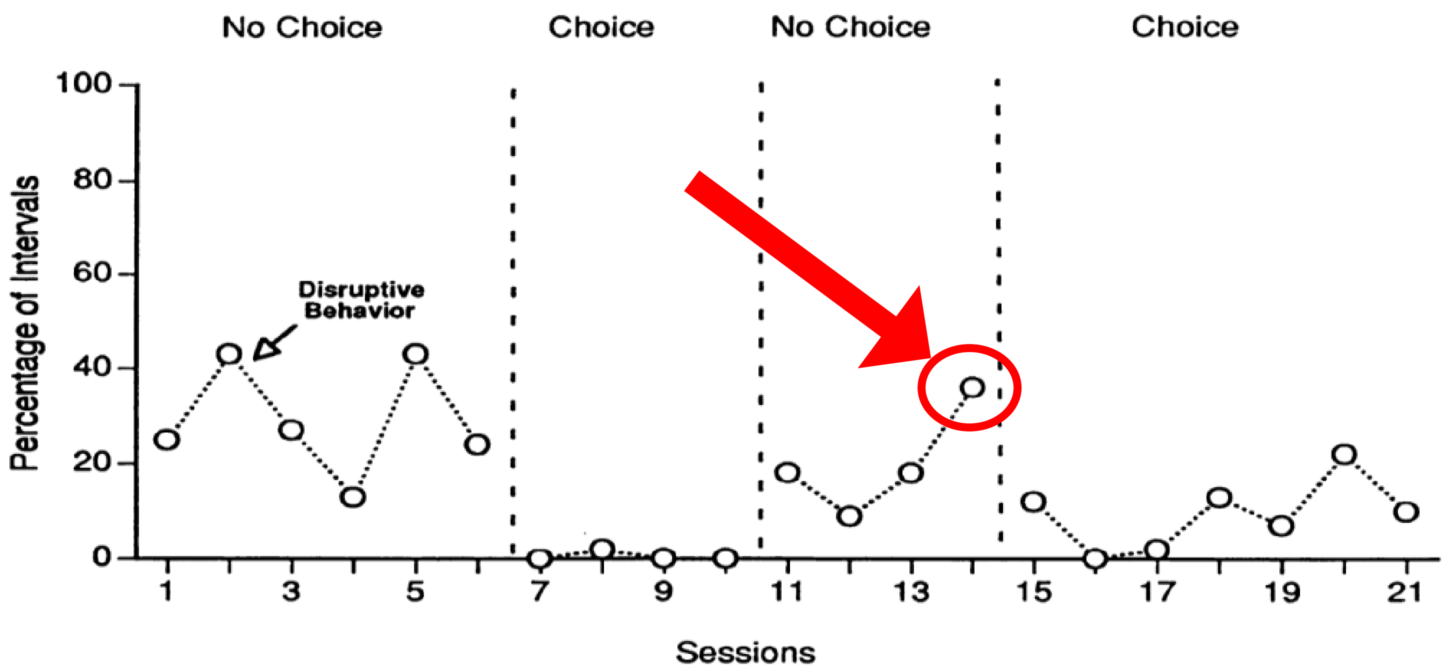
Measurement procedure	# Cases
Interval recording	19
Continuous recording	5
Event counting	3
Momentary time sampling	1
Other	4

Another approach to defining effect sizes

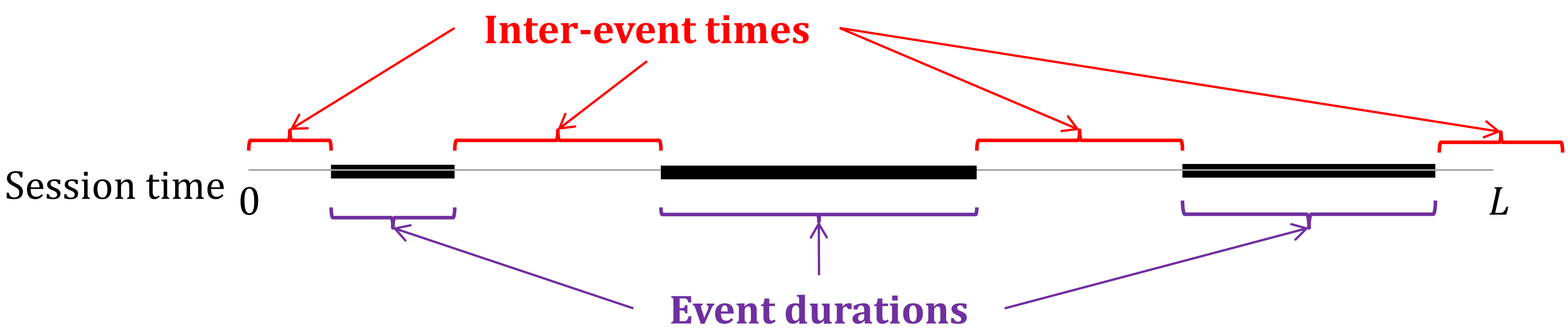
- **Free-operant behavior:** behavior that can occur at any time, without prompting or restriction by the investigator (e.g., disruptive behavior, physical aggression, motor stereotypy, smiling, slouching).
- **Prevalence**: the proportion of time that a behavior occurs
- **The prevalence ratio**: $$\frac{(\text{Prevalence during treatment})}{(\text{Prevalence during baseline})}$$
- Why?
  - Prevalence is most relevant dimension of free-operant behavior.
  - Captures how single-case researchers talk about their results.
  - Empirical fit.

A model for free-operant behavior

- Session-level measurement model.



A model for free-operant behavior



Alternating Renewal Process (Rogosa & Ghandour, 1991)

1. Event durations are identically distributed, with average duration $\mu > 0$.
2. Inter-event times (IETs) are identically distributed, with average IET $\lambda > 0$.
3. Event durations and IETs are all mutually independent.
4. Process is in equilibrium.

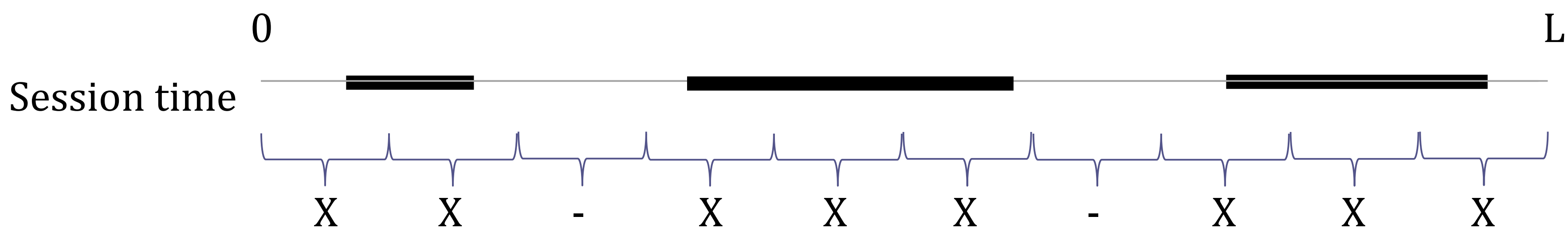
Continuous recording

1. Note beginning and end time of each event.
2. Find total duration of all events.
3. Divide by total session length: Y = (total duration) / L.

Continuous recording measures **prevalence.** Under ARP,

$$E(Y) = \frac{\mu}{\mu + \lambda}$$

Partial interval recording



1. Divide session into *K* short intervals, each of length *P*.
2. During each interval, note whether behavior occurs at all.
3. Calculate proportion of intervals where behavior occurs:

Y = (# Intervals with behavior) / K.

Partial interval recording (cont.)

- Partial interval recording does not measure prevalence (Altmann, 1974; Kraemer, 1979).
- In an alternating renewal process,

$$E(Y) = \frac{\mu}{\mu + \lambda} + \beta \qquad \beta = \frac{\int_0^P \Pr(IET > x)dx}{\mu + \lambda} > 0$$

The prevalence ratio, defined

- Baseline phase: μ^B, λ^B
- Treatment phase: μ^T, λ^T
- The prevalence ratio:

$$\Omega = \frac{\mu^T / \left(\mu^T + \lambda^T\right)}{\mu^B / \left(\mu^B + \lambda^B\right)}$$

- "No effect" corresponds to $\Omega = 1$.
- Confidence intervals, meta-analysis on natural log scale.

$$\log \Omega = \log\left(\frac{\mu^T}{\mu^T + \lambda^T}\right) - \log\left(\frac{\mu^B}{\mu^B + \lambda^B}\right)$$

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Estimation

- Continuous recording
  (and other measurement procedures)
  - Conventional methods: generalized linear models.

- Partial interval data
  - Need to invoke additional assumptions.

Partial interval data: Bounding the bias

- Pick a value μ_{min} where you are certain that $\mu^B, \mu^T > \mu_{min}$.
- Then, under ARP,

$$\Omega^L \leq \Omega \leq \Omega^U$$

where

$$\Omega^L \equiv \frac{E\left(Y^T\right)}{E\left(Y^B\right)} \times \left(\frac{\mu_{min}}{\mu_{min} + P}\right) \qquad \Omega^U \equiv \frac{E\left(Y^T\right)}{E\left(Y^B\right)} \times \left(\frac{\mu_{min} + P}{\mu_{min}}\right)$$

Y^B outcome in baseline phase
Y^T outcome in treatment phase

Partial interval data: Bounding the bias (cont.)

- Estimate the bounds with sample means.

$$\hat{\Omega}^L \equiv \frac{\bar{y}_T}{\bar{y}_B} \times \left(\frac{\mu_{min}}{\mu_{min} + P} \right) \qquad \hat{\Omega}^U \equiv \frac{\bar{y}_T}{\bar{y}_B} \times \left(\frac{\mu_{min} + P}{\mu_{min}} \right)$$

$\bar{y}_B$ sample mean in baseline phase, $\bar{y}_T$ sample mean in treatment phase

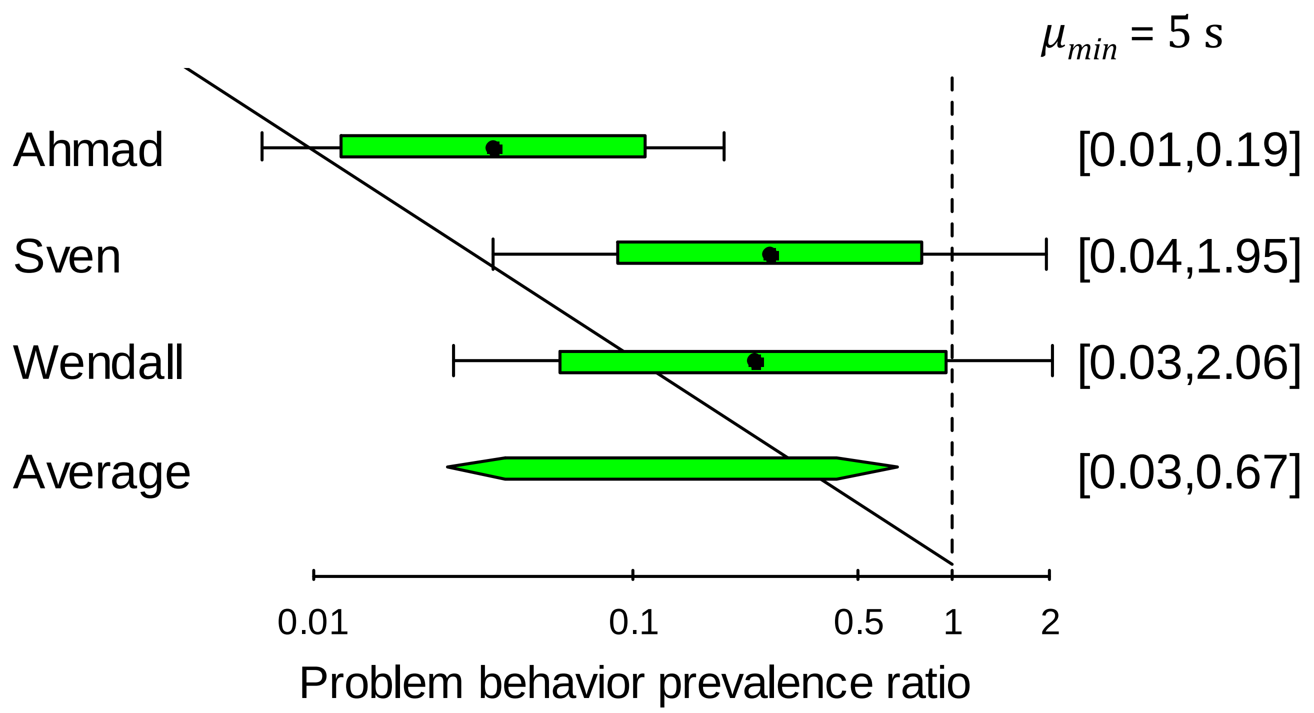
- With approximate variance (on log-scale)

$$Var\left(\log \hat{\Omega}^L\right) = Var\left(\log \hat{\Omega}^U\right) \approx \frac{s_T^2}{n_T \left(\bar{y}_T\right)^2} + \frac{s_B^2}{n_B \left(\bar{y}_B\right)^2}$$

s_B^2 sample variance in baseline phase, s_T^2 sample variance in treatment phase
n_B observations in baseline phase, n_T observations in treatment phase

Dunlap, et al. (1994)

Choice making to promote adaptive behavior for students with emotional and behavioral challenges.



Partial interval data: Other strategies

What assumptions are necessary to get point estimates?

1. Distributional assumption about IETs, plus no change in mean duration.
2. Distributional assumptions about IETs and event durations.

Conclusion

- Limit scope to a specific class of outcomes.
- Use a model to
  - Separate operational definitions from estimation procedures.
  - Address comparability of different outcome measurement procedures.
- Emphasize assumptions justifying estimation procedures.

Future directions

- Other effect size proposals in light of free-operant model
- Improved measurement procedures?
  - Combining momentary time sampling & interval recording
- Design comparability
  - Hedges, Pustejovsky, & Shadish (2012) on standardized mean differences

Thank you

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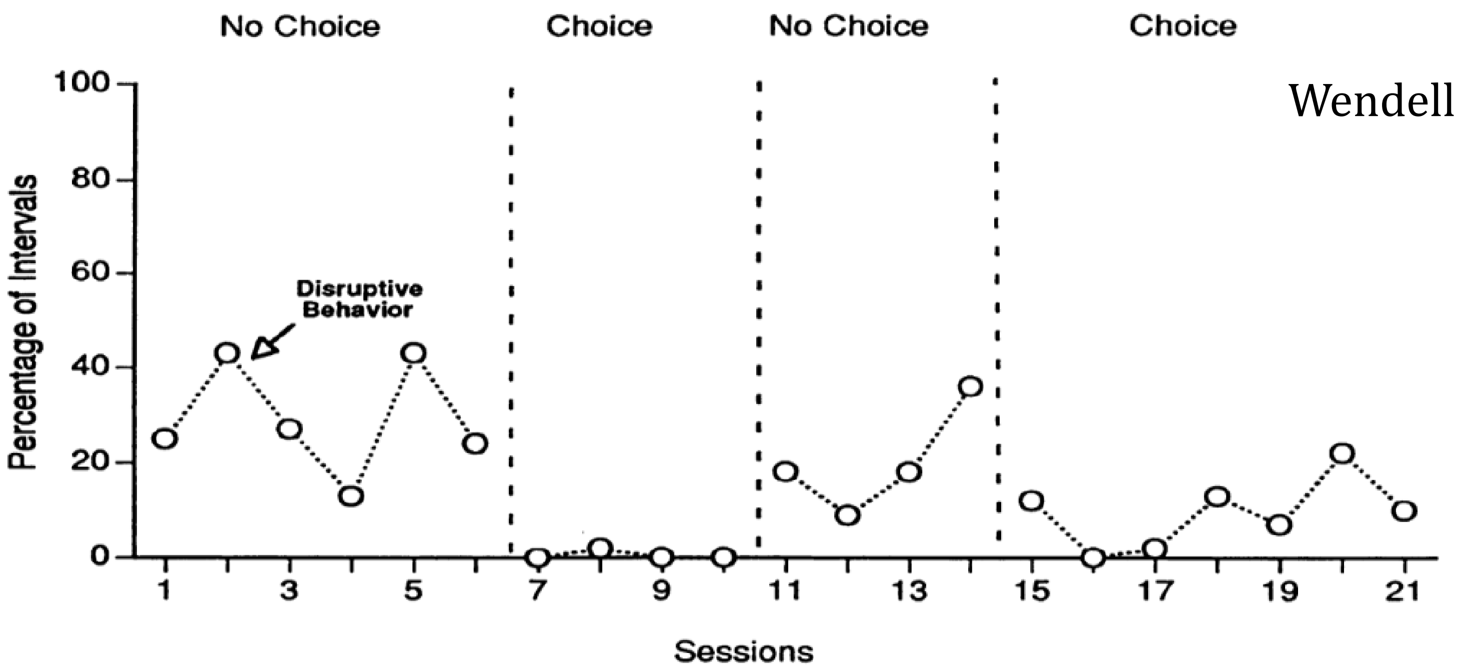
- Larry Hedges
- Will Shadish
- Kristynn Sullivan
- Jonathan Boyajian

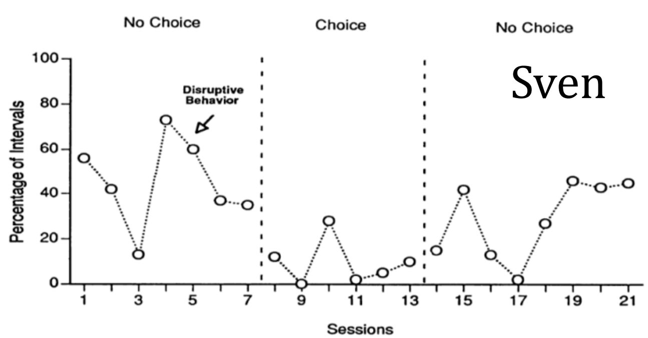
Extras

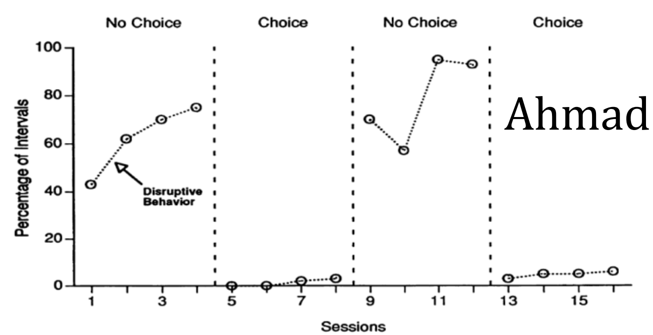
- Notation
- Other effect sizes
  - In the literature
  - For free-operant behavior
- Outcome classes in single-case research
- Measurement procedures for free-operant behavior
- Shogren meta-analysis results
- Estimation
  - Continuous recording
  - Partial interval recording (Strategy 2, Strategy 3, Strategy 4)
- Generalized linear models

Dunlap, et al. (1994)

Choice making to promote adaptive behavior for students with emotional and behavioral challenges.







Notation & definitions

- L = length of observation session
- μ = average event duration
  - μ^B during baseline phase
  - μ^T during treatment phase
- λ = average inter-event time
  - λ^B during baseline phase
  - λ^T during treatment phase
- Ω = Prevalence ratio

$$\Omega = \frac{\mu^T / (\mu^T + \lambda^T)}{\mu^B / (\mu^B + \lambda^B)}$$

- β = Bias of partial interval data where P is the interval length.

$$\beta = \frac{\int_0^P \Pr(IET > x)dx}{\mu + \lambda}$$

Effect sizes for single-case designs

- Standardized Mean Difference (Busk & Serlin, 1992)

$$SMD \equiv \frac{\bar{y}_T - \bar{y}_B}{s_{pool}} \qquad s^2_{pool} = \frac{(n_B - 1)s_B^2 + (n_T - 1)s_T^2}{n_B + n_T - 2}$$

- Mean Baseline Reduction (Campbell & Herzinger, 2010)

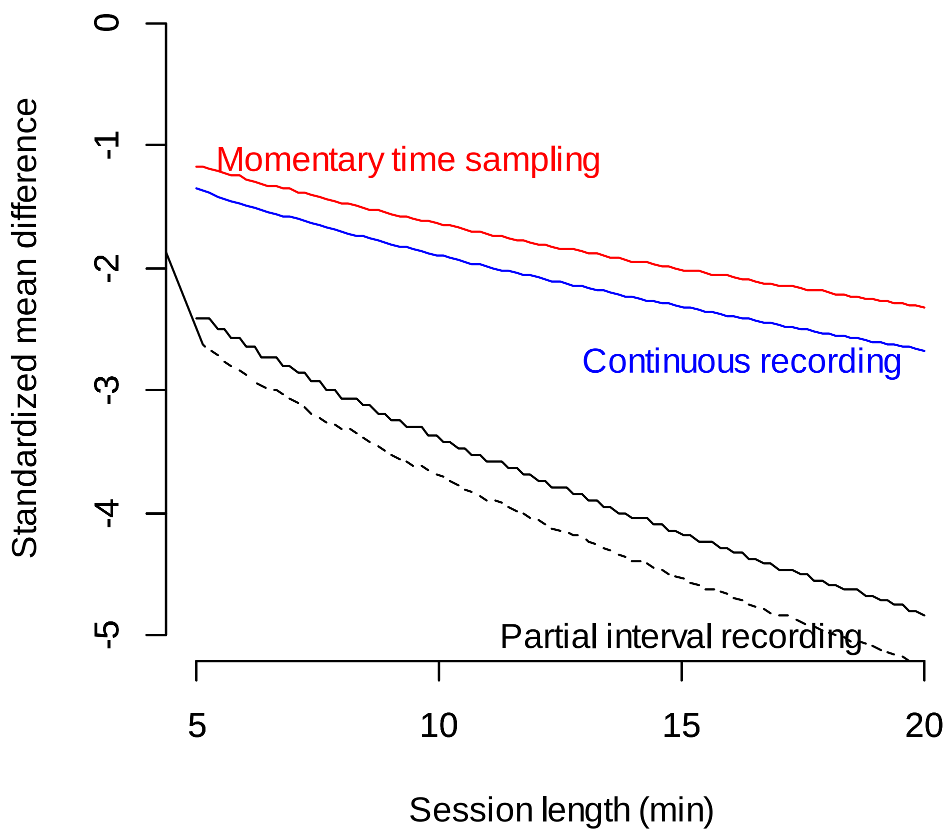
$$MBR \equiv \frac{\bar{y}_T - \bar{y}_B}{\bar{y}_B} \times 100\%$$

- Non-overlap of All Pairs (Parker & Vannest, 2009)

$$NAP \equiv \sum_{b:Trt_b = 0} \sum_{t:Trt_t = 1} \frac{I(Y_t < Y_b)}{n_T n_B}$$

Standardized mean difference

- Alternating Poisson Process with prevalence ratio = 0.5, $\mu^B = 20$ s, $\lambda^B = 20$ s, $\mu^T = 20$ s, $\lambda^T = 60$ s.



Possible effect sizes for free-operant behavior

$\dfrac{\mu_1}{\mu_0}$	Duration Ratio
$\dfrac{\lambda_0}{\lambda_1}$	Inter-Event Time Ratio
$\dfrac{\mu_0 + \lambda_0}{\mu_1 + \lambda_1}$	Incidence Ratio
$\dfrac{\mu_1 / (\mu_1 + \lambda_1)}{\mu_0 / (\mu_0 + \lambda_0)}$	Prevalence Ratio
$\dfrac{\mu_1 / \lambda_1}{\mu_0 / \lambda_0}$	Prevalence Odds Ratio

Outcomes in single-case research

Outcome	% of Studies
Free-operant behavior	56
Restricted-operant behavior	41
Academic	8
Physiological/psychological	6
Other	3

N = 122 single-case studies published in 2008, as identified by Shadish & Sullivan (2011).

- **Restricted-operant** behavior occurs in response to a specific stimulus, often controlled by the investigator.
- **Free-operant** behavior can occur at any time, without prompting or restriction by the investigator (e.g., physical aggression, motor stereotypy, smiling, slouching).

Free-operant behavior

- Physical aggression
- Nail biting
- Smiling
- Tics
- Motor stereotypy
- Initiating social interaction
- Maintaining proper posture

Measurement procedures for free-operant behavior

Recording procedure	% of Studies
Event counting	60
Interval recording	19
Continuous recording	10
Momentary time sampling	7
Other	16

N = 68 single-case studies measuring free-operant behavior, a subset of all 122 studies published in 2008, as identified by Shadish & Sullivan (2011). Characteristics of single-case designs used to assess intervention effects in 2008. *Behavior Research Methods*, *43*(4), 971–80.

Event counting

1. Count the number of events that occur during the session.
2. Divide by session length to get rate of events per unit time: Y = (# events) / L.

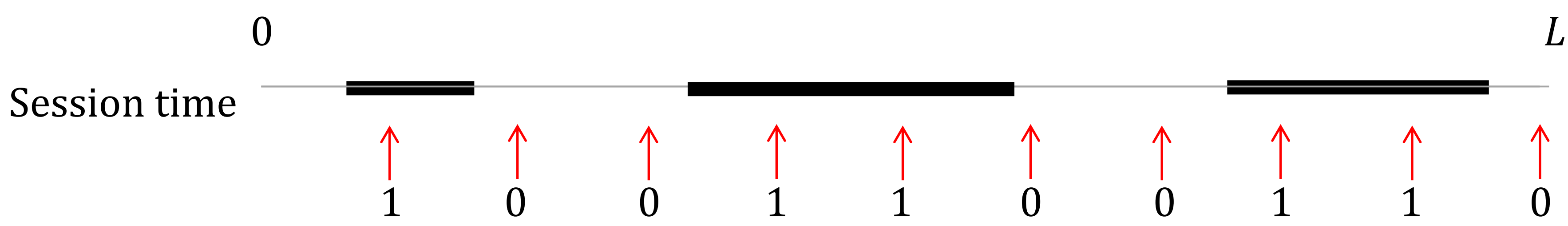
Under ARP, event counting measures **incidence**[1]

$$E(Y) = \frac{1}{\mu + \lambda} > 0$$

1. Cox, D. R. (1962). *Renewal Theory*, p. 46.

Momentary time sampling

1. Divide session into *K* short intervals.
2. At end of each interval, note whether behavior is occurring *at that moment*.
3. Calculate proportion of moments where behavior occurs: Y = (# moments with behavior) / *K*.



Under ARP, momentary time sampling measures **prevalence**[1]

$$E(Y) = \frac{\mu}{\mu + \lambda}$$

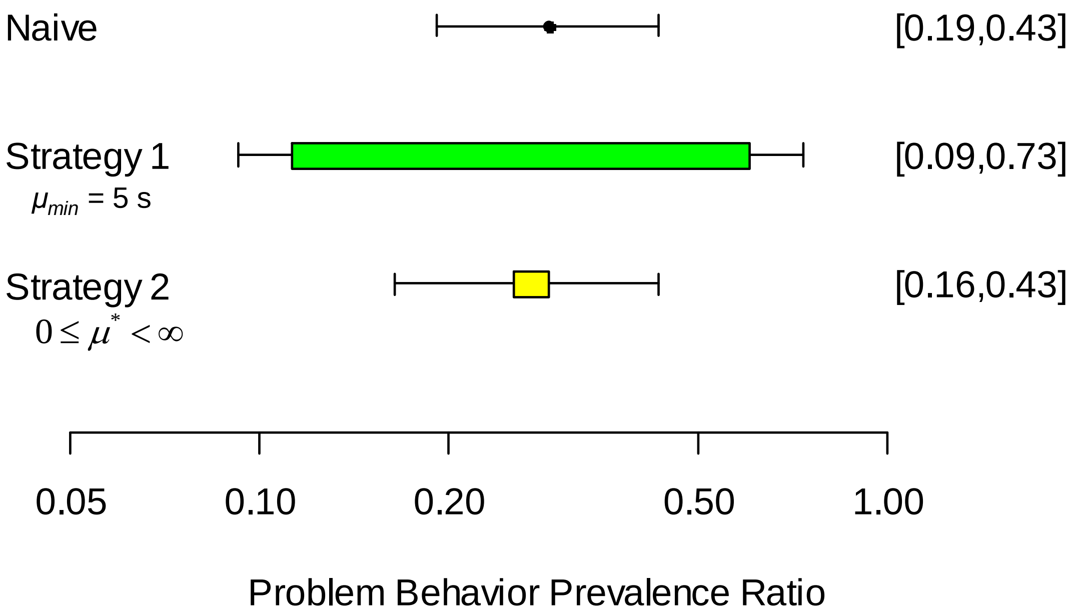
1. Cox, D. R. (1962). *Renewal Theory*, p. 86.

Observation recording procedures for free-operant behavior

Procedure	Measure	Expectation
Event counting	Incidence	$\frac{1}{\mu + \lambda}$
Continuous recording	Prevalence	$\frac{\mu}{\mu + \lambda}$
Momentary time sampling	Prevalence	$\frac{\mu}{\mu + \lambda}$
Partial interval recording	Neither prevalence nor incidence	$\frac{\mu}{\mu + \lambda} + \beta$

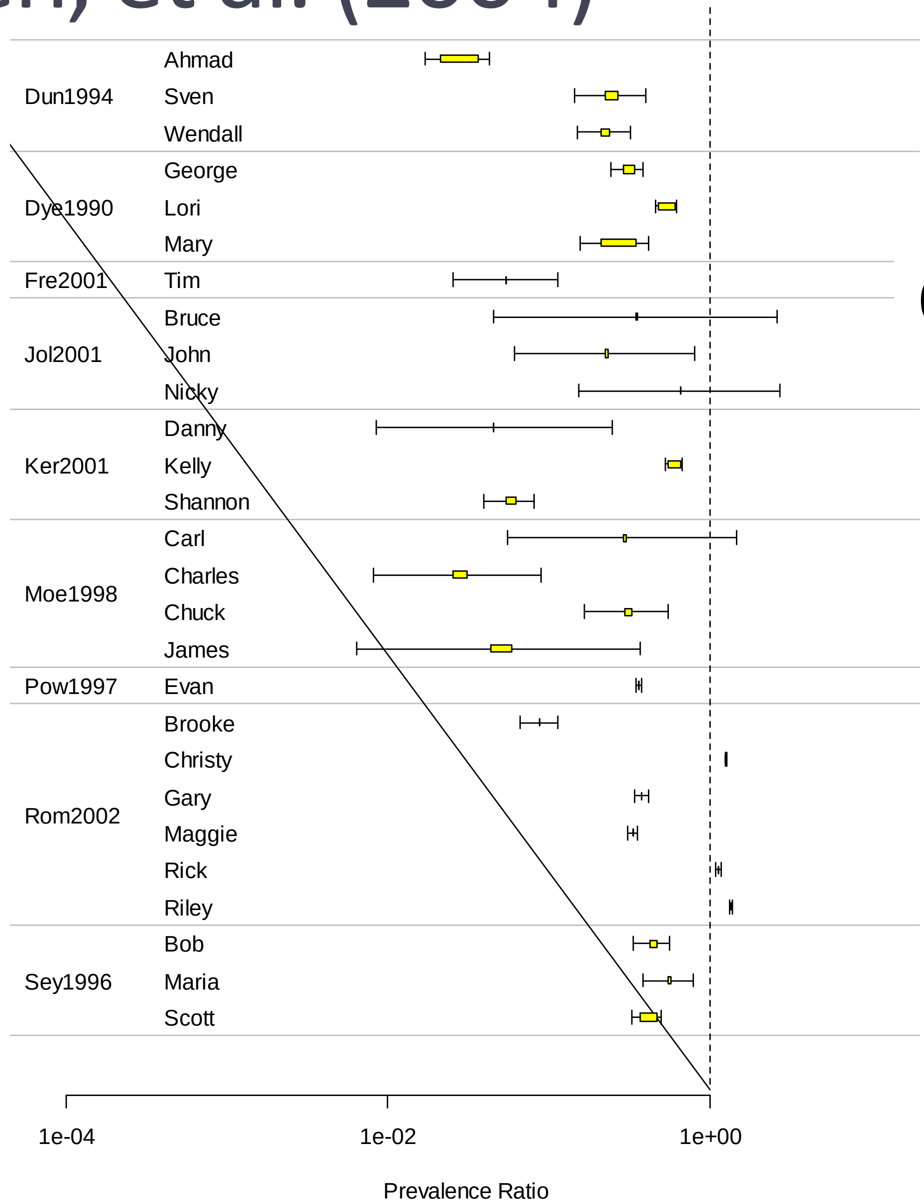
Shogren (2004) meta-analysis

The effect of choice-making as an intervention for problem behavior.



The most misleading assumptions are the ones you don't even know you're making.
- Douglas Adams, Last Chance to See

Shogren, et al. (2004)



$$0 \leq \mu^{*} < \infty$$

Session-level model for one case, continuous recording data

The data:

- n sessions, n^B in baseline and n^T in treatment phase(s)
- Y_j outcome measurement for j^{th} session
- Trt_j covariate indicating if session is in a treatment phase

The case-level model:

- Constant prevalence within each phase.

$$\omega = \log E\left(Y_j \mid Trt_j = 1\right) - \log E\left(Y_j \mid Trt_j = 0\right)$$

Effect size estimation: Continuous recording

- A basic moment estimator:

$$\hat{\omega} = \log(\bar{y}_T) - \log(\bar{y}_B)$$

$$\bar{y}_B = \frac{1}{n^B}\sum_{j=1}^{n^B} Y_j\left(1 - Trt_j\right) \qquad \bar{y}_T = \frac{1}{n^T}\sum_{j=n^B+1}^{n^B+n^T} Y_j Trt_j$$

- Its approximate variance:

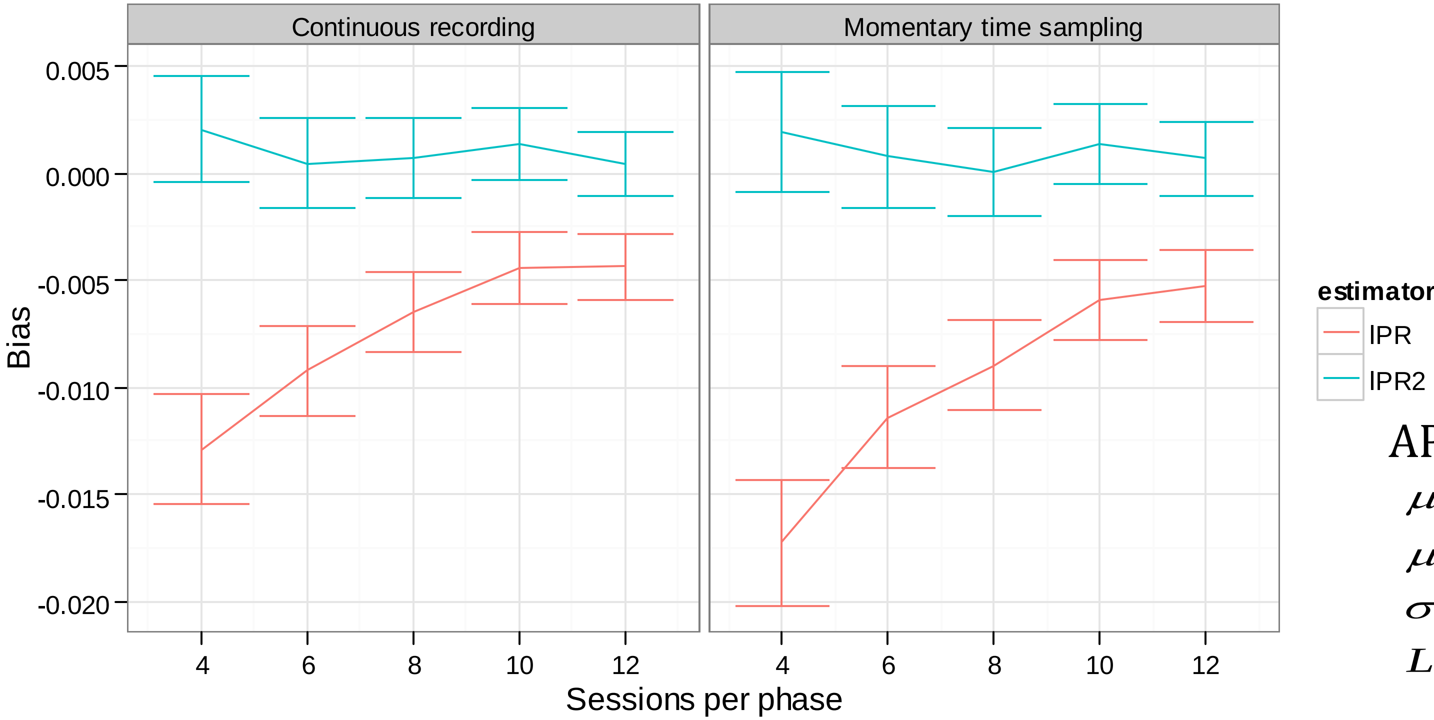
$$Var(\hat{\omega}) \approx \frac{s_T^2}{n_T\left(\bar{y}_T\right)^2} + \frac{s_B^2}{n_B\left(\bar{y}_B\right)^2}$$

$$s_B^2 = \frac{1}{n^B - 1}\sum_{j=1}^{n^B}\left(1 - Trt_j\right)\left(Y_j - \bar{y}_B\right)^2 \qquad s_T^2 = \frac{1}{n^T - 1}\sum_{j=n^B+1}^{n^B+n^T} Trt_j\left(Y_j - \bar{y}_T\right)^2$$

Effect size estimation: Continuous recording

- A bias-corrected estimator:

$$\hat{\omega}_2 = \ln(\bar{y}_T) + \frac{s_T^2}{2n_T(\bar{y}_T)^2} - \ln(\bar{y}_B) - \frac{s_B^2}{2n_B(\bar{y}_B)^2}$$



APP model with

$\mu^B = 20, \lambda^B = 20$

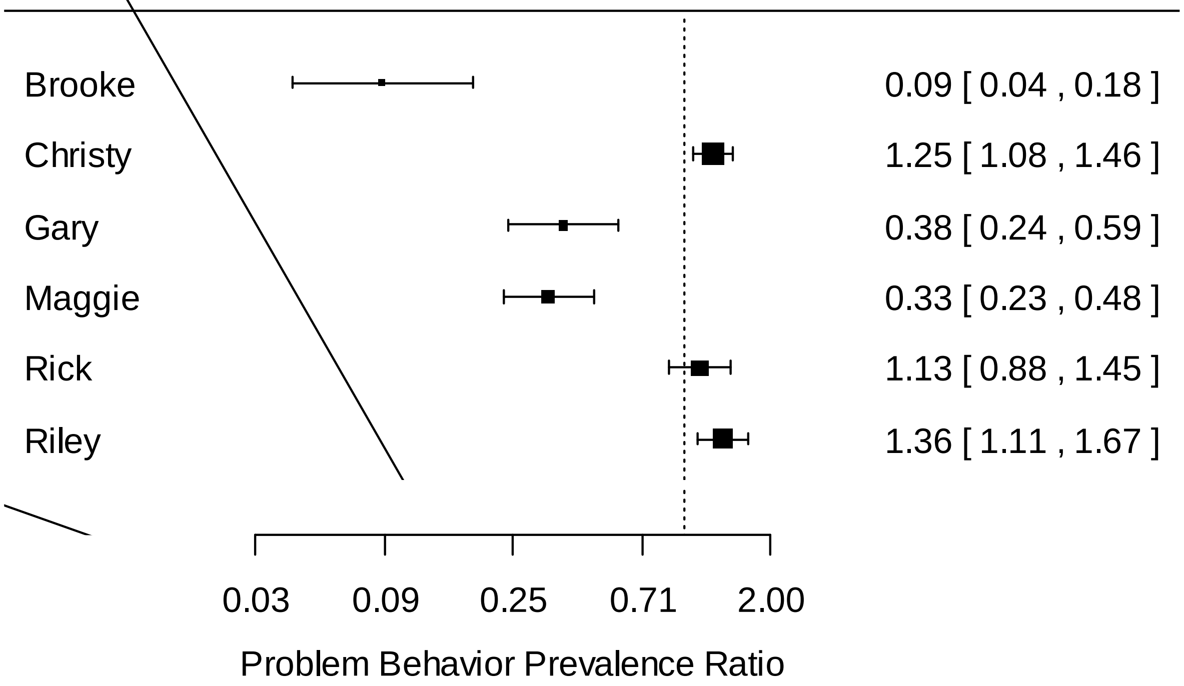
$\mu^T = 20, \lambda^T = 60$

$\sigma_\epsilon = 0.15, \phi = 0.5,$

$L = 600, K = 20$

Romaniuk, et al. (2002)

The influence of activity choice on problem behaviors maintained by escape versus attention.



"Students who displayed attention-maintained problem behavior did not show any effects as a result of the choice intervention."

Effect size estimation: Partial interval data

Strategy 2: Point estimate

Assumptions:

1. IETs are exponentially distributed.
2. Average duration is constant across phases: $\mu^B = \mu^T$.
3. Assume that $\mu^B = \mu^T = \mu^*$, for some known μ^*.

Effect size estimation: Partial interval data

Strategy 2: Point estimate (cont.)

- Find estimates for λ^B and λ^T by solving

$$\bar{y}_B = 1 - \hat{\lambda}^B e^{-P/\hat{\lambda}^B} / \left(\mu^* + \hat{\lambda}^B\right) \qquad \bar{y}_T = 1 - \hat{\lambda}^T e^{-P/\hat{\lambda}^T} / \left(\mu^* + \hat{\lambda}^T\right)$$

- Estimate Ω with

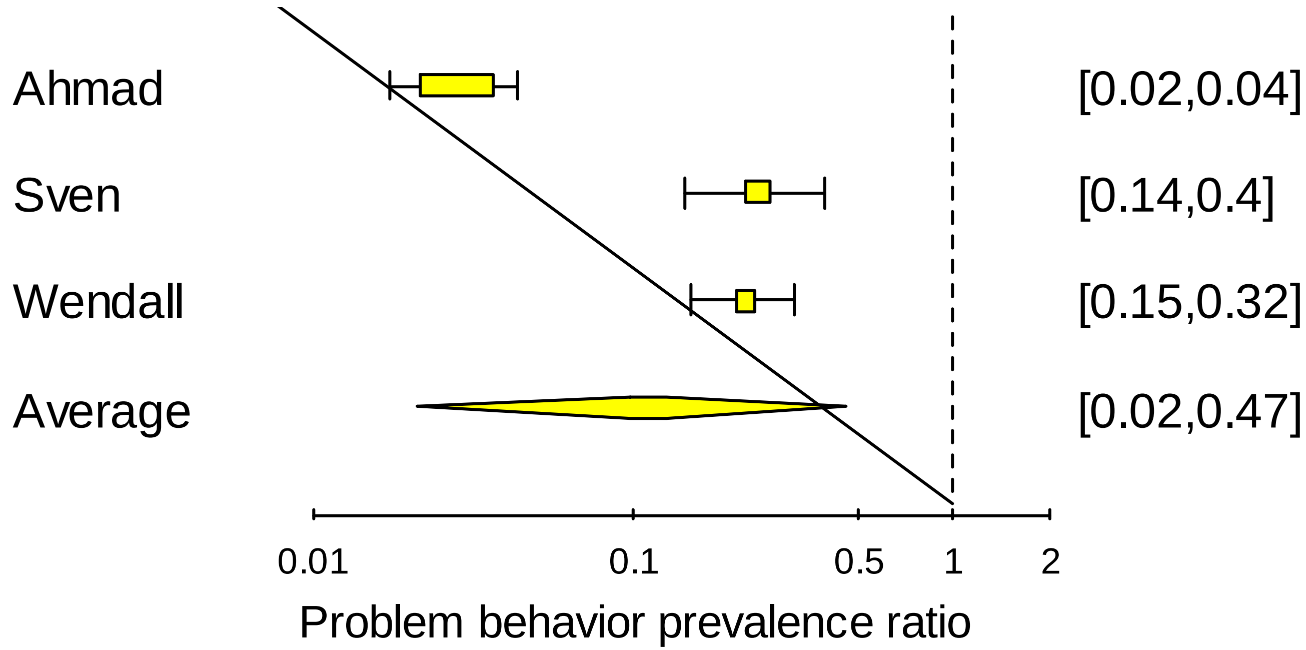
$$\hat{\Omega} = \frac{\mu^* / \left(\mu^* + \hat{\lambda}^T\right)}{\mu^* / \left(\mu^* + \hat{\lambda}^B\right)}$$

$$Var\left(\log \Omega\right) \approx \sum_{p=B,T} \frac{\left(\hat{\lambda}^p\right)^4 s_p^2}{\left(1 - \bar{y}_p\right)^2 \left[\mu^* \hat{\lambda}^p + P\left(\mu^* + \hat{\lambda}^p\right)\right]^2}$$

Dunlap, et al. (1994) again

Choice making to promote adaptive behavior for students with emotional and behavioral challenges.

$$0 \leq \mu^* < \infty$$



Effect size estimation: Partial interval data

Strategy 3: Parametric bound

- Assume that IETs are exponentially distributed.
- Assume that $\mu^B = \mu^T$.
- If $E(Y^T) < E(Y^B)$ then $\omega^L \leq \omega \leq \omega^U$

$$\omega^L = \ln\left[-\ln E\left(1 - Y^B\right)\right] - \ln\left[-\ln E\left(1 - Y^T\right)\right]$$

$$\omega^U = \ln\left[E\left(Y^T\right)\right] - \ln\left[E\left(Y^B\right)\right]$$

- Estimate the bounds with sample means.

$$\hat{\omega}^L = \ln\left[-\ln\left(1 - \bar{y}_B\right)\right] - \ln\left[-\ln\left(1 - \bar{y}_T\right)\right]$$

$$\hat{\omega}^U = \ln\left(\bar{y}_T\right) - \ln\left(\bar{y}_B\right)$$

Effect size estimation: Partial interval data

Strategy 4: Moment estimation

- Assume that IETs are exponentially distributed.
- Assume that event durations are exponentially distributed.
- Estimate μ, λ in each phase by setting sample mean & variance equal to moment functions:

$$\bar{y}^T = E\left(Y^T\right) = f\left(\mu^T, \lambda^T\right)$$

$$s_T^2 = Var\left(Y^T\right) = g\left(\mu^T, \lambda^T\right)$$

- Disadvantages:
  - Assumes no extra session-to-session variability
  - Need to know number of intervals
  - Large sampling variability

Generalized linear models

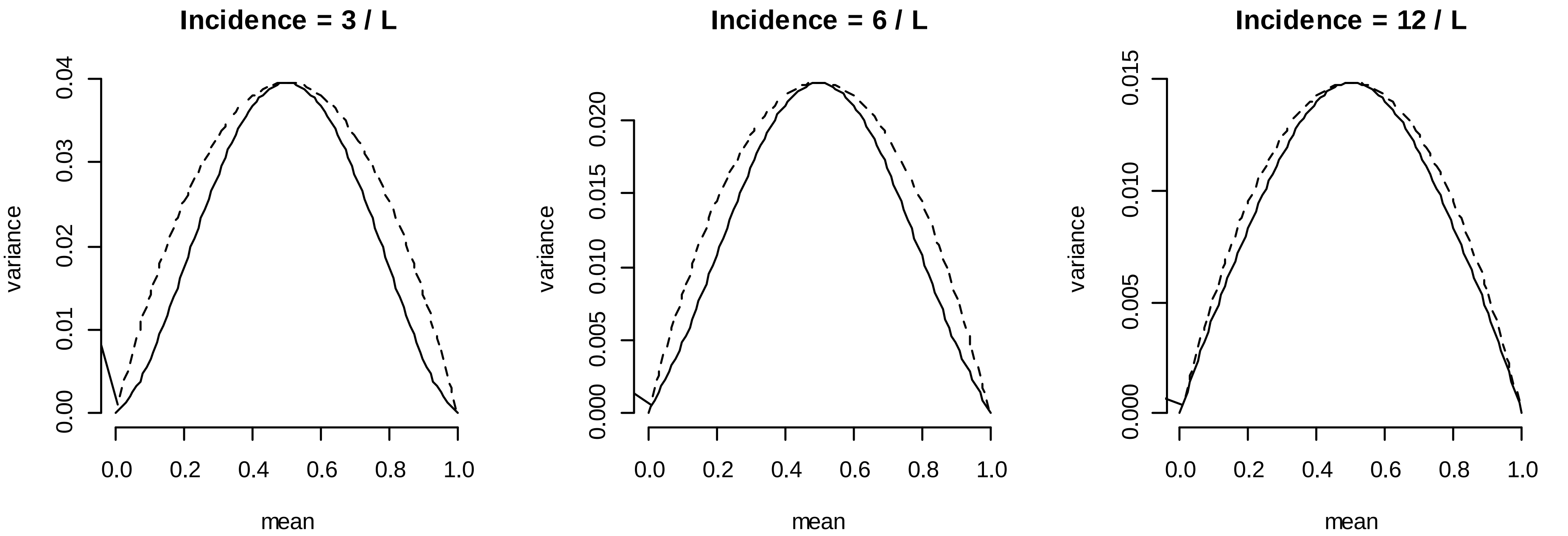
- Relationship between mean and linear regression?
  - log link function

$$\ln E\left(Y_j\right) = \beta_0 + \beta_1 Trt_j$$

  - logit link for prevalence odds ratio
- Relationship between mean and variance?

Relationship between mean and variance?

- Under APP, with momentary time sampling, the relationship between the mean and the variance depends on the behavior's incidence.



Variance functions

Recording procedure	**Variance under APP model**
Continuous recording	$Var(Y) = \frac{2\phi^2(1-\phi)^2}{IL} + \frac{1-\exp[-IL/\phi(1-\phi)]}{I^2L^2}$
Momentary time sampling	$Var(Y) = \frac{\phi(1-\phi)}{K}\left[1 + \frac{2}{K}\sum_{k=1}^{K}(K-k)\exp\left(\frac{-ILk}{K\phi(1-\phi)}\right)\right]$
Event counting	$Var(Y) = IL\left[\phi^2 + (1-\phi)^2\right] + 2\phi^2(1-\phi)^2\left[1 - \exp\left(\frac{-IL}{\phi(1-\phi)}\right)\right]$

$$\phi = \frac{\mu}{\mu+\lambda}, \quad I = \frac{1}{\mu+\lambda}$$

"Ballpark" variance function

- Relationship between mean and variance?
  - "Ballpark" variance function
  - Sandwich variance estimation (heteroscedasticity robust)

Recording procedure	Ballpark variance function
Continuous recording	$V(x) = x^2(1-x)^2$
Momentary time sampling	$V(x) = x\,(1 - x)$
Event counting	$\mathrm{V}(x) = x$

Adding serial dependence

- Regression with serial dependence between sessions:

$$\ln E\left(Y_j \mid \epsilon_j\right) = \beta_0^* + \beta_1 Trt_j + \epsilon_j$$

where $(\epsilon_1, \epsilon_2, \ldots, \epsilon_n)$ follow an auto-regressive model

- Dealing with serial dependence
  - Ignore it, use empirical variance estimates for meta-analysis
  - Estimate it to improve precision of point estimates