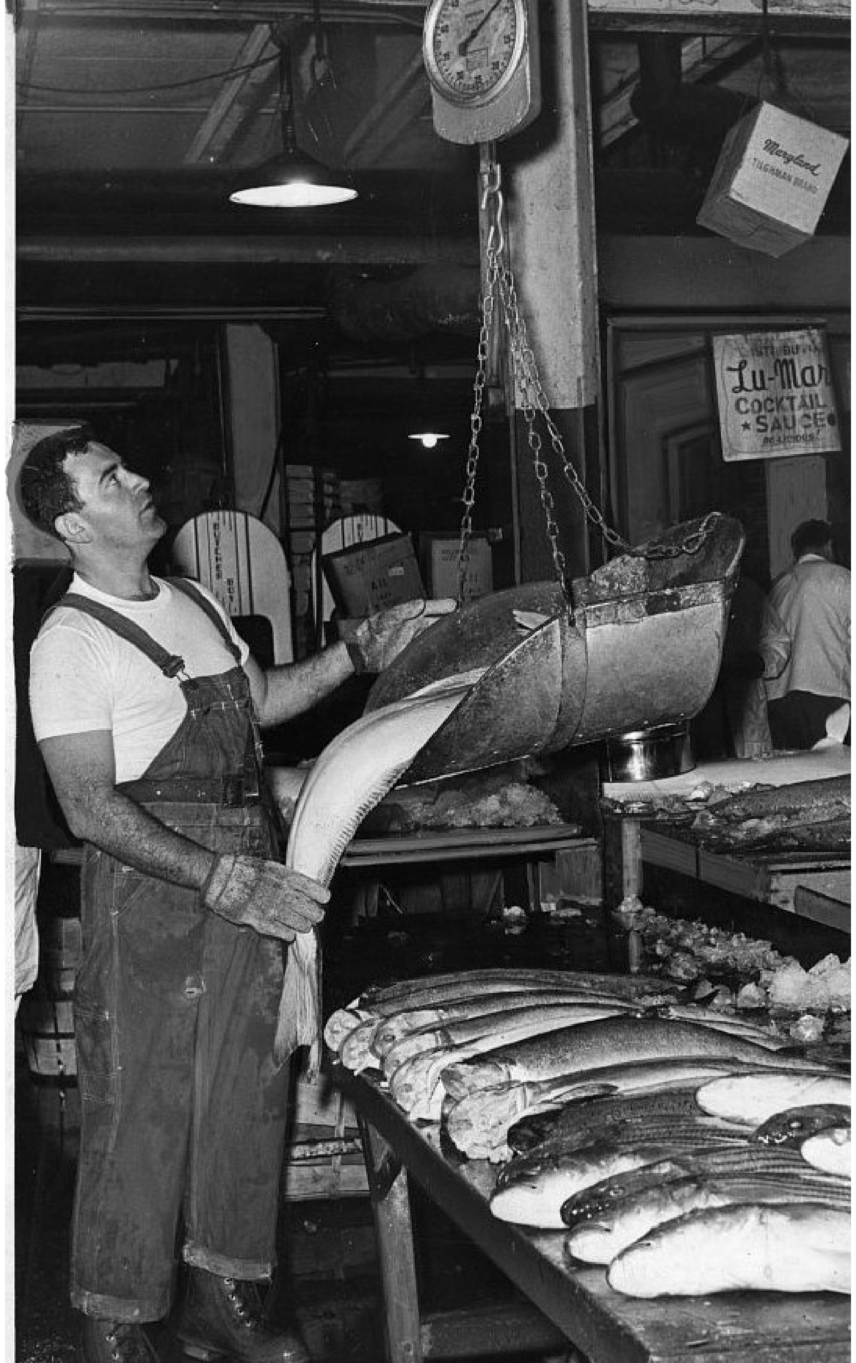



# A matter of emphasis: Comparison of working models for meta-analysis of dependent effect sizes

## James E. Pustejovsky

### Society for Research Synthesis Methodology

### July 20, 2022, Portland, OR / Online

WISCONSIN
UNIVERSITY OF WISCONSIN–MADISON

# Dependent effect size estimates

### Multiple outcomes measured on a common set of participants

| | |
|---|---|
| Treatment | O P Q |
| Control | O P Q |

$d_{O1}$
$d_{P1}$
$d_{Q1}$

### Outcomes measured at multiple follow-up times

| | | | |
|---|---|---|---|
| Treatment | $O_1$ | $O_2$ | $O_3$ |
| Control | $O_1$ | $O_2$ | $O_3$ |

$d_{12}$
$d_{22}$
$d_{32}$

### Multiple treatment conditions compared to a common control

| | |
|---|---|
| Treatment T | O |
| Treatment U | O |
| Control | O |

$d_{T3}$
$d_{U3}$

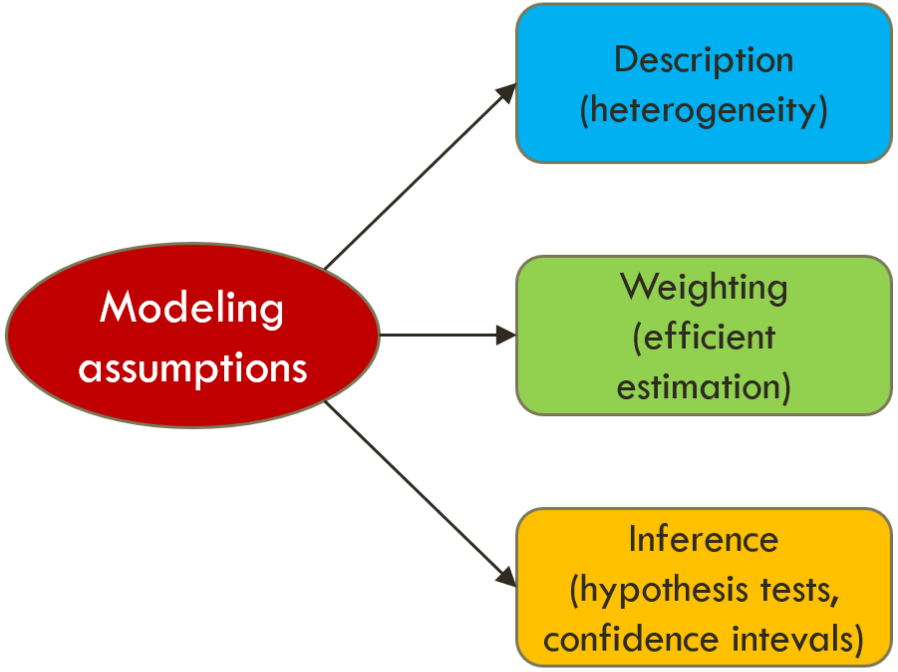
### Multiple correlations from a common sample

| | a | b | c | d |
|---|---|---|---|---|
| a | | | | |
| b | | | | |
| c | | | | |
| d | | | | |

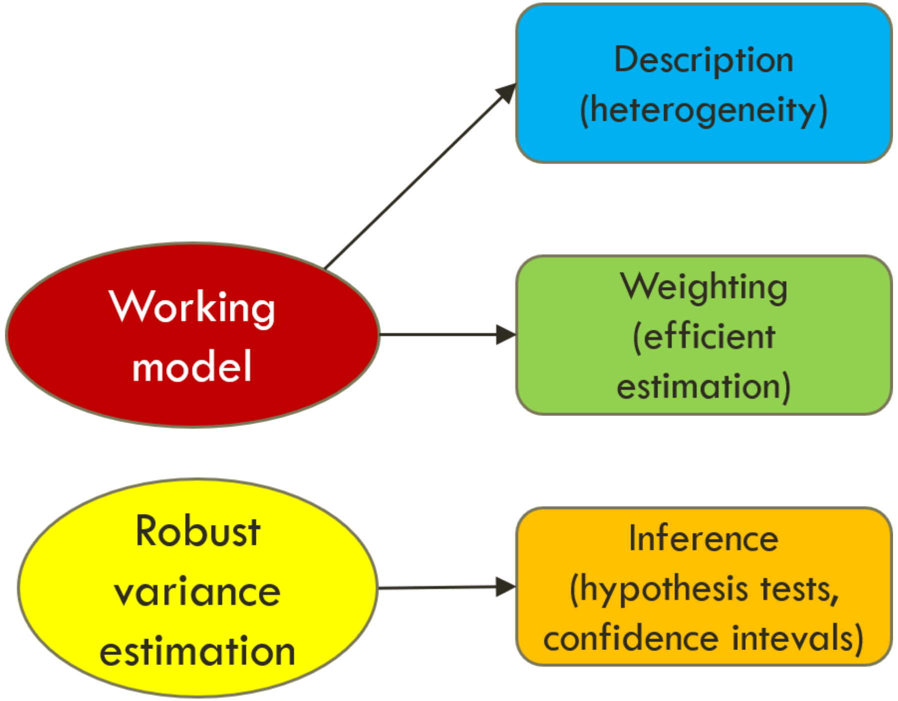
$r_{ab}$
$r_{ac}$
$r_{ad}$

# From modeling assumptions to working models

## Conventional approach




## Robust variance estimation



# Many available working models

- **Correlated Effects** model (Hedges, Tipton, & Johnson, 2010) implemented in `robumeta`
- **Hierarchical Effects** model (Hedges, Tipton, & Johnson, 2010) implemented in `robumeta`
- **Multi-level meta-analysis** (MLMA) as a working model (Van den Noortgate et al., 2013, 2015; Fernandez-Castilla et al., 2020)
- **Correlated-and-Hierarchical Effects** working model (Pustejovsky & Tipton, 2020)
- **Independent effects** (i.e., a basic random effects model)

**Which working model(s) should be used in practice?**

**How much does this choice matter?**

**Why might results based on different working models differ?**

# Meta-regression with study-level covariates

- Meta-analysis with $J$ studies
- Study $j$ includes $k_j$ effect size estimates
- Effect size estimate $T_{ij}$, sampling standard error $\sigma_j$
- Study-level predictors $\mathbf{x}_j$

- Meta-regression model:

$$T_{ij} = \mathbf{x}_j \boldsymbol{\beta} + \epsilon_{ij}$$

- Different working models make different assumptions about $\epsilon_{ij}$'s.
    - $\tau^2$ between-study variance
    - $\omega^2$ within-study variance
    - $\rho$ assumed sampling correlation between effect size estimates

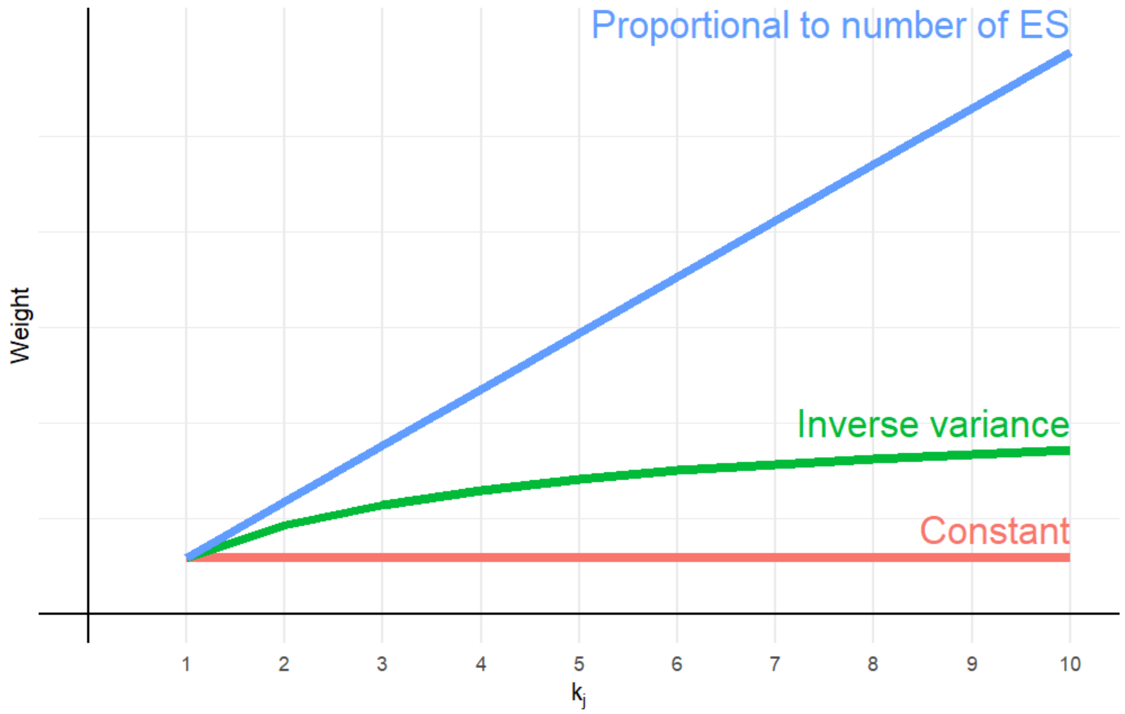
- An equivalent study-level regression:

$$\bar{T}_j = \mathbf{x}_j \boldsymbol{\beta} + \bar{\epsilon}_j$$

where different working models assign different weights to each study.

# A matter of emphasis

- Working models differ in how weight is allocated to studies with different $k_j$'s.




- Results from different working models will differ only if $E\left(\bar{T}_j | k_j, \mathbf{x}_j\right)$ depends on $k_j$.

# Approximate working weights

- Independent effects
  - $w_j$ proportional to $k_j$

$$w_j \approx \frac{k_j}{\tau^2 + \omega^2 + \sigma_j^2}$$

- Hierarchical effects (robumeta)
  - $w_j$ proportional to $k_j$

$$w_j \approx \frac{k_j}{\tau^2 + \rho \times g + \omega^2 + \sigma_j^2} \qquad (g \text{ small})$$

- Correlated effects (robumeta)
  - $w_j$ does not depend on $k_j$

$$w_j \approx \frac{1}{\tau^2 + \omega^2 f + \sigma_j^2} \qquad (f > 1)$$

- Correlated-and-hierarchical effects:
  - Inverse-variance $w_j$

$$w_j \approx \frac{k_j}{k_j \tau^2 + (k_j - 1)\rho \sigma_j^2 + \omega^2 + \sigma_j^2}$$

- Multi-level meta-analysis:
  - Nearly inverse-variance $w_j$

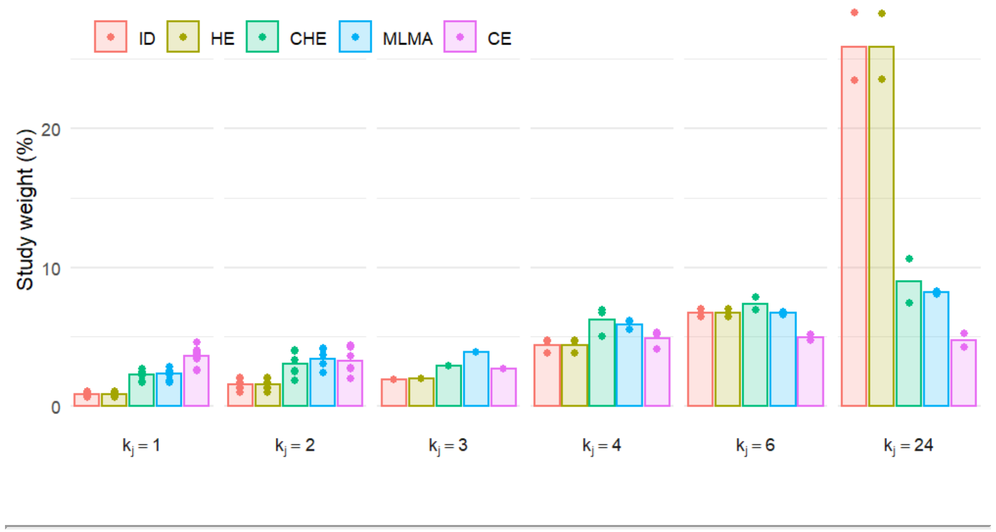
$$w_j \approx \frac{k_j}{k_j(\tau^2 + h) + (\omega^2 - h) + \sigma_j^2}$$

# Action video game effects on cognitive performance

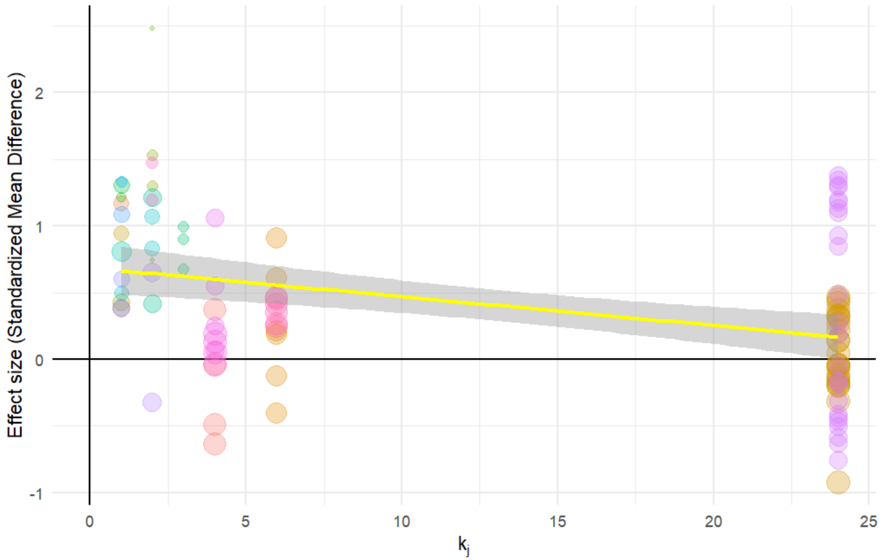
- Bediou and colleagues (2018) reported a synthesis of experimental studies examining the effects of playing action video games on cognitive performance.
- 26 studies, 99 standardized mean difference effect size estimates
  - $k_j$ ranging from 1 to 24, median = 2.
- Sensitivity analysis across working models:

| Model | Est (SE) | 95% CI | $\tau^2$ | $\omega^2$ | $\tau^2 + \omega^2$ |
|---|---|---|---|---|---|
| Independent Effects | 0.33 (0.11) | [0.07, 0.60] | - | - | 0.17 |
| Hierarchical Effects | 0.33 (0.11) | [0.07, 0.60] | 0.05 | 0.13 | 0.18 |
| Correlated Effects | 0.62 (0.09) | [0.43, 0.82] | 0.13 | - | 0.13 |
| Correlated + Hierarchical Effects | 0.51 (0.10) | [0.30, 0.72] | 0.01 | 0.22 | 0.23 |
| Multi-Level Meta-Analysis | 0.55 (0.10) | [0.34, 0.76] | 0.11 | 0.10 | 0.21 |

## Weight allocation by working model



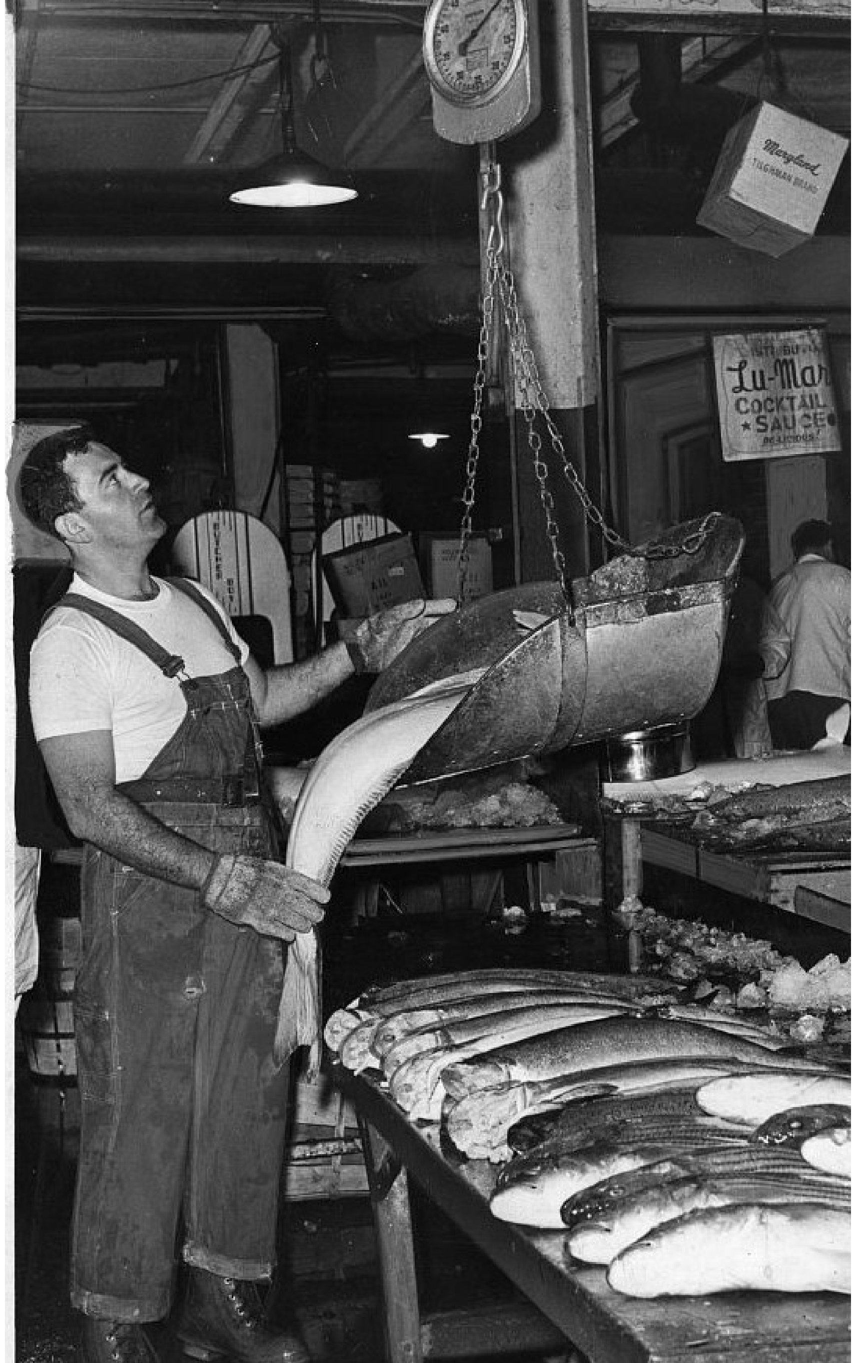

## Effect sizes are correlated with $k_j$



# Some tentative implications

For meta-regression with study-level covariates...

- Working model sensitivity arises when a) $k_j$'s vary and b) effect sizes are correlated with $k_j$'s.
  - Report distribution of $k_j$'s!
  - Perhaps better to assess $\mathrm{E}\left(\bar{T}_j | k_j, \mathbf{x}_j\right)$ directly?
  - Consider reasons that $\mathrm{E}\left(\bar{T}_j\right)$ varies with $k_j$ (selective reporting? overly lenient inclusion criteria?).
- The original correlated effects and hierarchical effects working models entail extreme, polar opposite weighting schemes.
  - Using either as primary working model warrants careful justification.
- We need to pay more attention to within-study heterogeneity of effects.




# Thanks

pustejovsky@wisc.edu

https://jepusto.com

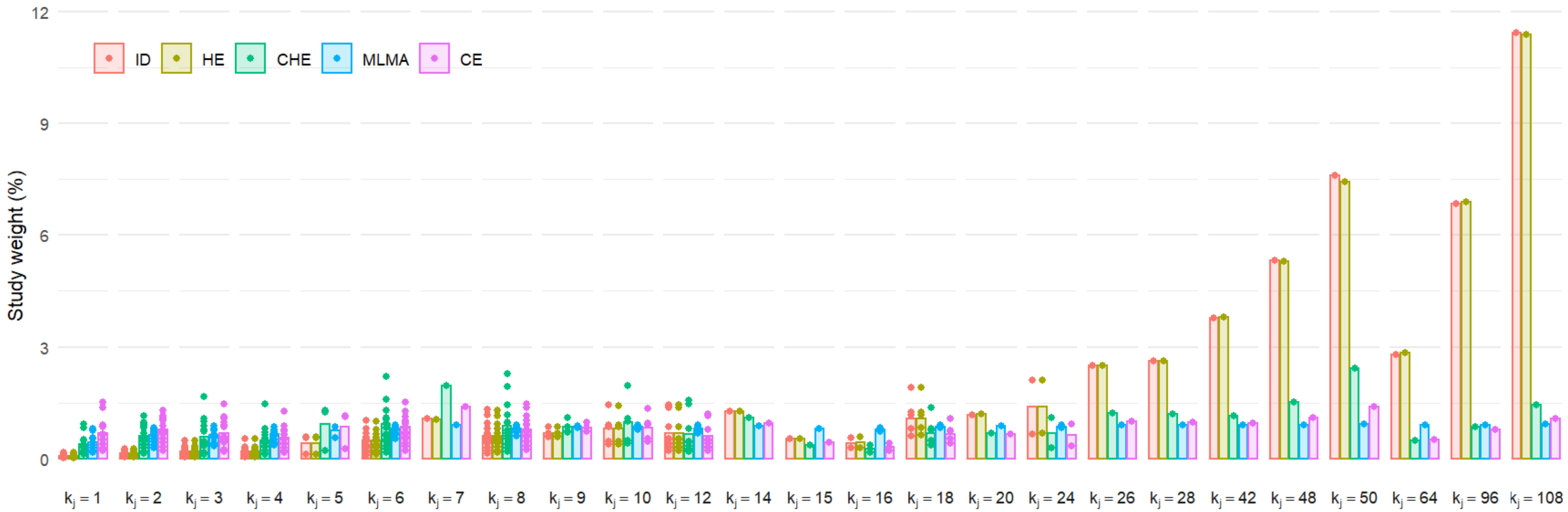
WISCONSIN
UNIVERSITY OF WISCONSIN–MADISON

# Effects of Brief Alcohol Interventions

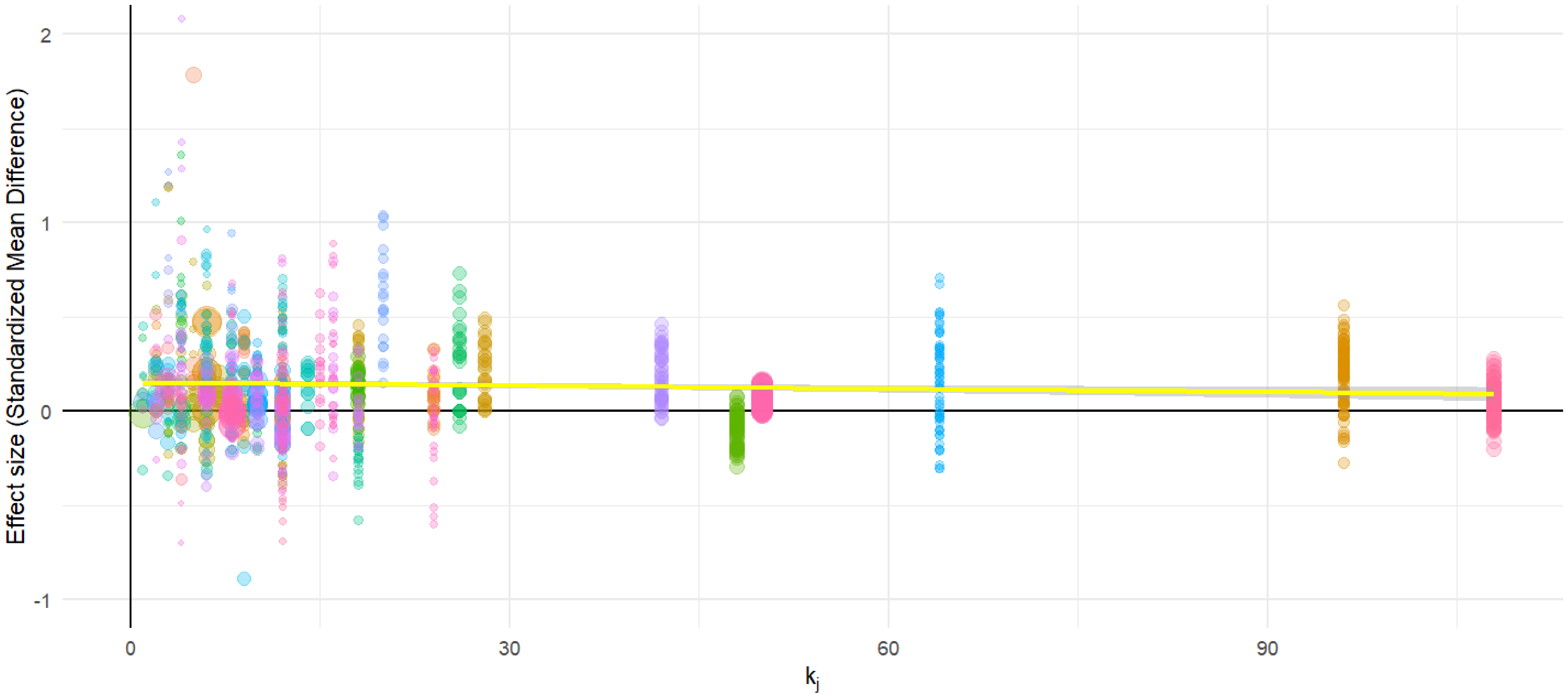
- Tanner-Smith and Lipsey (2015) reported a synthesis of experimental studies examining the effects of brief alcohol interventions to reduce alcohol consumption in adolescents and young adults.
- 137 studies, 1333 standardized mean difference effect size estimates
    - $k_j$ ranging from 1 to 108, median = 6.
- Sensitivity analysis across working models:

| Model | Est (SE) | 95% CI | $\tau^2$ | $\omega^2$ | $\tau^2 + \omega^2$ |
|---|---|---|---|---|---|
| Independent Effects | 0.11 (0.02) | [0.07, 0.15] | - | - | 0.02 |
| Hierarchical Effects | 0.11 (0.02) | [0.07, 0.15] | 0.02 | 0.00 | 0.02 |
| Correlated Effects | 0.13 (0.02) | [0.10, 0.16] | 0.03 | - | 0.03 |
| Correlated + Hierarchical Effects | 0.12 (0.02) | [0.09, 0.15] | 0.01 | 0.02 | 0.02 |
| Multi-Level Meta-Analysis | 0.15 (0.02) | [0.11, 0.19] | 0.04 | 0.00 | 0.04 |

# Weight allocation by working model



# Effect sizes are *not* correlated with $k_j$



# Working model weights (estimated)

- Independent effects

$$w_j = \frac{k_j}{\breve{\omega}^2 + \sigma_j^2}$$

- Hierarchical effects (robumeta)

$$w_j = \frac{k_j}{\ddot{\tau}^2 + \ddot{\omega}^2 + \sigma_j^2}$$

- Correlated effects (robumeta)

$$w_j = \frac{1}{\dot{\tau}^2 + \sigma_j^2}$$

- Correlated-and-hierarchical effects:

$$w_j = \frac{k_j}{k_j \hat{\tau}^2 + k_j \rho \sigma_j^2 + \hat{\omega}^2 + (1-\rho)\sigma_j^2}$$

- Multi-level meta-analysis:

$$w_j = \frac{k_j}{k_j \tilde{\tau}^2 + \tilde{\omega}^2 + \sigma_j^2}$$