What are we modeling? Using predictive fit to inform effect metric choice in meta-analysis

James E. Pustejovsky

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Research Synthesis

> The systematic integration of empirical results across **multiple sources of evidence**, for purposes of drawing generalizations[1].

Meta-Analysis

> Statistical models and methods to support quantitative research synthesis.

Fields that rely on research synthesis

- Medicine (Cochrane Collaboration)
- Education (What Works Clearinghouse)
- Psychology
- Social policy (justice, welfare, public health, etc.)
- Economics, international development
- Ecology and Environmental Science
- Physical sciences

- Some background on meta-analysis
- The problem of effect metric choice
- Proposal: Use predictive fit criteria to inform metric choice
- Illustrations
- Discussion

Canonical Meta-Analysis

- We observe summary results from each of k studies:
  - T_i - effect size estimate
  - se_i - standard error of effect size estimate
  - $N_i, \mathbf{x}_i$ - sample size, other study features

- A summary random effects model:

$$T_i \sim N\left(\theta_i,\ se_i^2\right)$$
$$\theta_i \sim N\left(\mu,\ \tau^2\right)$$

- A random effects meta-regression:

$$T_i \sim N\left(\theta_i,\ se_i^2\right)$$
$$\theta_i \sim N\left(\mathbf{x}_i\boldsymbol{\beta},\ \tau^2\right)$$

- "Conceptual unity of statistical methods" for meta-analysis[2] suggests that most any effect size measure θ_i can be used, as long as $T_i \dot{\sim} N\left(\theta_i,\ se_i^2\right)$.

Prediction Interval

- An approximate $1 - 2\alpha$ prediction interval for a new study-specific parameter θ_{new}[3]:

$$\hat{\mu} \ \pm \ q_{\alpha} \times \sqrt{\hat{\tau}^2 + \mathbb{V}\left(\hat{\mu}\right)}$$

  - Largely used to characterize the extent of effect heterogeneity[4].

- Beyond this, “predictive modeling” culture[5,6] seems to have very little influence on meta-analysis.

Effect Metric Menagerie

Effect Metric Families

Single-group summaries

- Raw proportions π
- Arcsine-transformation $a = \text{asin}\left(\sqrt{\pi}\right)$
- Raw means μ

Bivariate associations / psychometric

- Pearson's correlation ρ
- Fisher's z-transformation $\zeta = \text{atanh}(\rho)$
- Cronbach's α coefficients (or transformations thereof)

Group comparison of binary outcomes

- Risk differences $\pi_1 - \pi_0$
- Risk ratios (log-transformed) $\log\left(\frac{\pi_1}{\pi_0}\right)$
- Odds ratios (log-transformed) $\log\left(\frac{\pi_1/(1-\pi_1)}{\pi_0/(1-\pi_0)}\right)$
- Bivariate models for π_0, π_1

Group comparison of continuous outcomes

- Raw mean differences $\mu_1 - \mu_0$
- Standardized mean differences $\delta = \frac{\mu_1-\mu_0}{\sigma}$
- Response ratios (log-transformed) $\lambda = \log\left(\frac{\mu_1}{\mu_0}\right)$
- Probability of superiority

Metric choice methodology

- Large literature on effect metrics for group comparison on binary outcomes.
  - Theoretical arguments about interpretability, stability, non-collapsibility[7,8].
  - Risk differences tend to be more heterogeneous[9,10].
- Strong opinions about effect metrics for group comparison on continuous outcomes[11].
  - Some novel alternatives to avoid standarization[12–14].
  - Various methods for standardization[e.g., 15,16].
- Choice between standardized mean difference and response ratio metrics
  - Sensitivity analyses using both metrics[17].
  - Model both metrics simultaneously[18].

Effect Metric Choice

- Choice of metric is constrained by
  - Studies designs
  - Data availability, reporting conventions
  - Heterogeneity of study features (e.g., outcome scales)

- Metric choice is driven by disciplinary conventions.
  - In many applications, more than one metric could apply.

Metric choice by predictive fit criteria

- Evaluate effect metrics by performance in **predicting summary data for a new study**.
  - Data vector $\mathbf{d}_i$ consisting of summary statistics used to compute effect size estimates.

- Use leave-one-out log-predictive density to measure predictive performance.

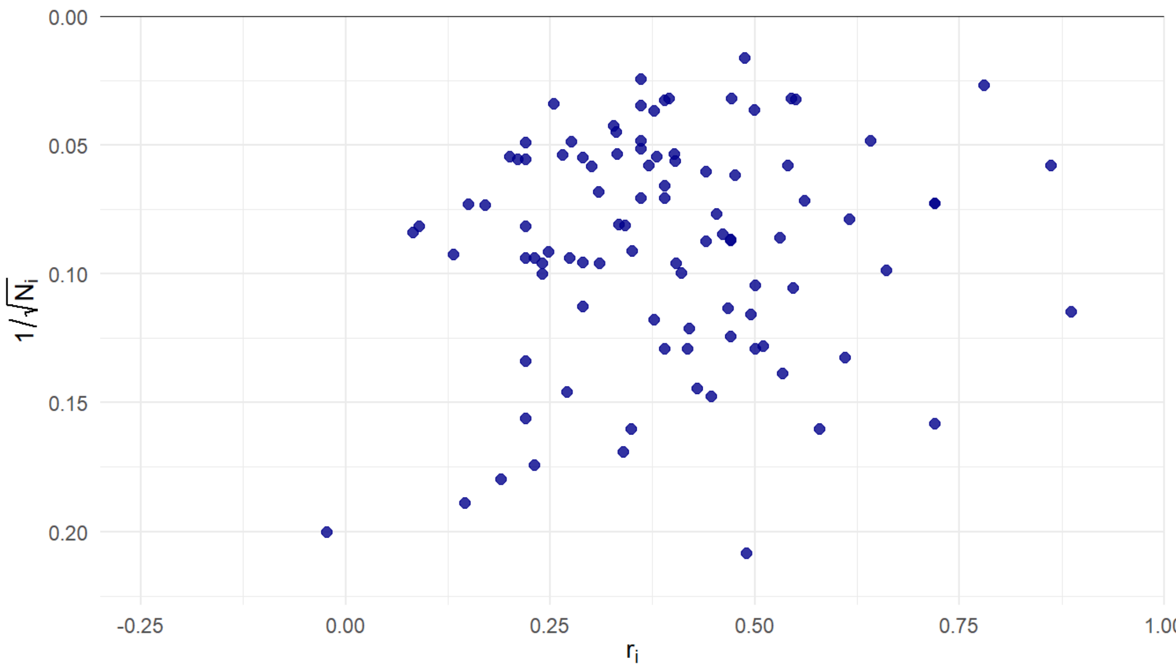
$$LPD = \frac{1}{k} \sum_{i=1}^{k} \log p \left(\mathbf{d}_i \,\middle|\, \hat{\mu}_{(-i)}, \hat{\tau}_{(-i)}, \mathbf{X}_i, N_i \right)$$

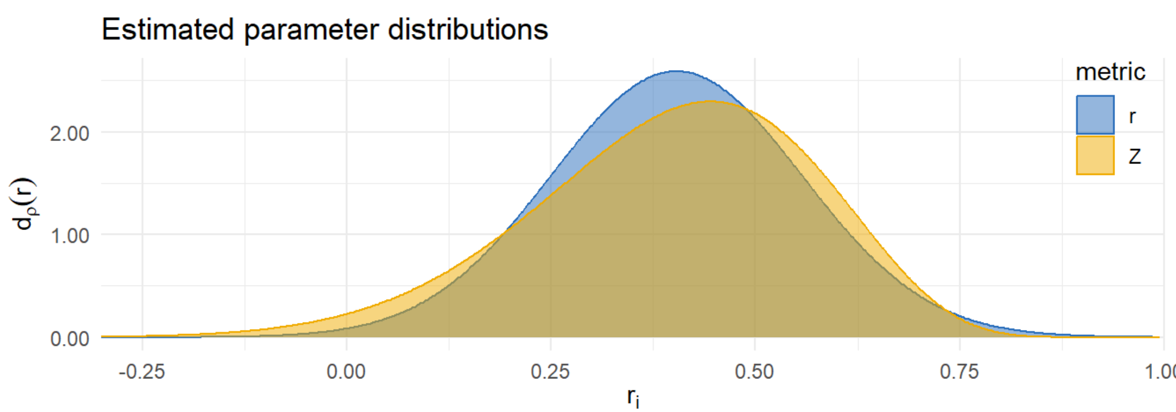
Two challenges

1. Polishing up models to generate predictions.
2. Conventional meta-analysis focuses on one-dimensional $f(\mathbf{d}_i)$, so we need auxiliary models for the rest of the data.

Class attendance and college grades

- Credé and colleagues[19] reported a systematic review and meta-analysis of studies on association between class attendance and grades / GPA in college.
- 99 correlation estimates, samples ranging from N_i = 23 to 3900 (median = 151, IQR = 76-335).





Bivariate associations

- The data: Pearson correlation between two variables of interest from a sample of N_i observations, r_i.

ρ metric

- Effect size estimate r_i, standard error $se_i = \frac{1-r_i^2}{\sqrt{N_i}}$
- Predictive model:

$$r_i \dot{\sim} N\left(\rho_i,\ \frac{(1-\rho_i^2)^2}{N_i}\right)$$
$$\rho_i \sim \ N_{trunc}\left(\mu_\rho,\ \tau_\rho^2\right)$$

$\zeta = \mathrm{atanh}(\rho)$ metric

- Effect size estimate $z_i = \mathrm{atanh}(r_i)$, standard error $se_i = \frac{1}{\sqrt{N_i - 3}}$
- Predictive model:

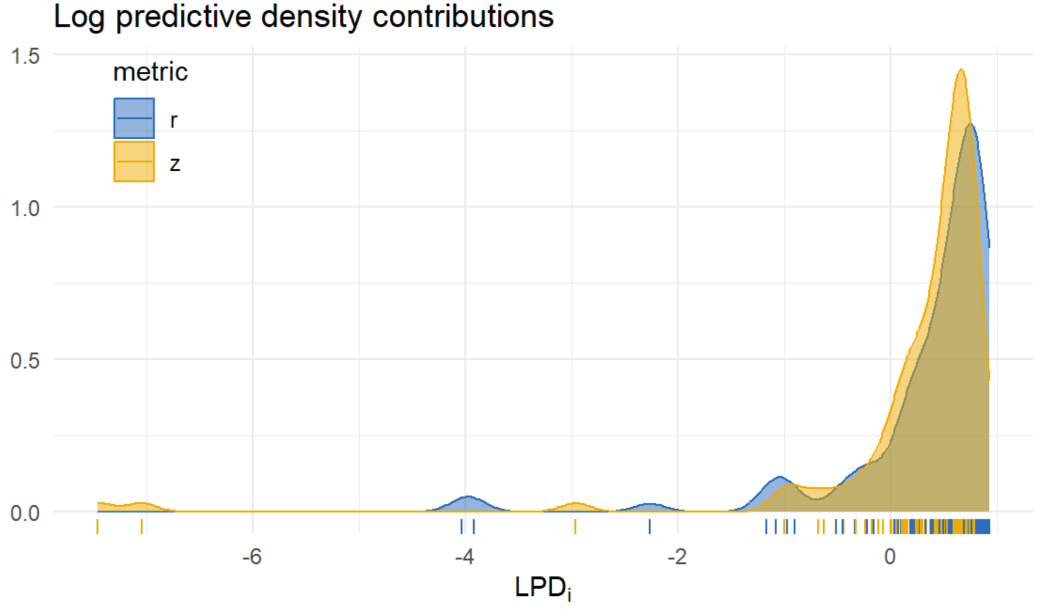
$$z_i \dot{\sim} N\left(\zeta_i,\ \frac{1}{N_i - 3}\right)$$
$$\zeta_i \sim \ N\left(\mu_\zeta,\ \tau_\zeta^2\right)$$

- log-predictive density:

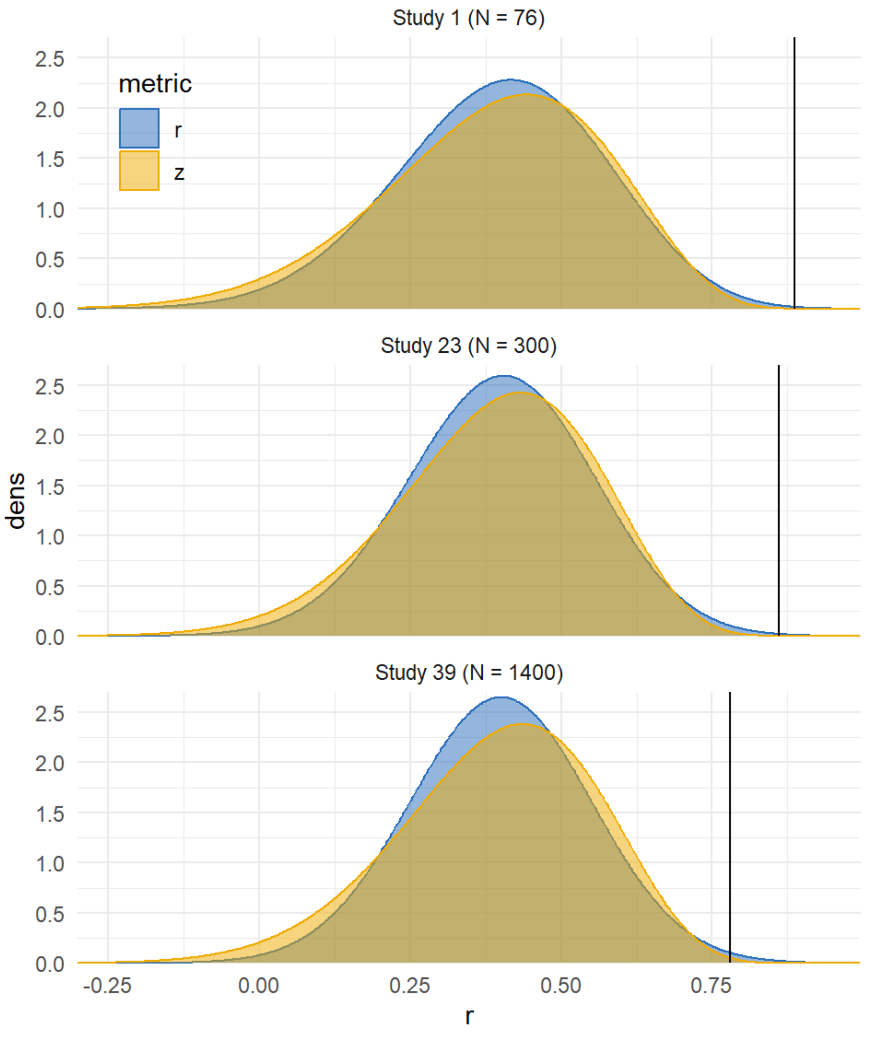
$$\begin{aligned}\log d_r \ &(r_i|\hat{\mu}_{\zeta(-i)}, \hat{\tau}_{\zeta(-i)}, N_i) \\ &= \log d_z\left(z_i \left|\hat{\mu}_{\zeta(-i)}, \hat{\tau}_{\zeta(-i)}, N_i\right.\right) - \log\left(1 - r_i^2\right)\end{aligned}$$

Metric comparison

Metric	Est.	95% CI	80% PI	LPD	SE
r	0.40	0.37-0.44	0.20-0.60	0.34	0.09
z	0.41	0.37-0.45	0.16-0.61	0.22	0.12
Difference				0.12	0.05

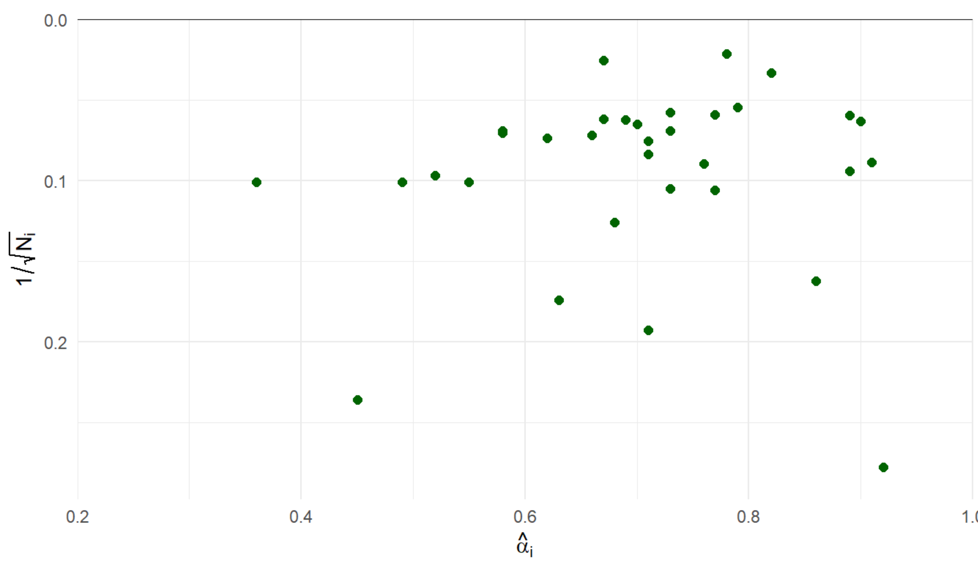


Outliers

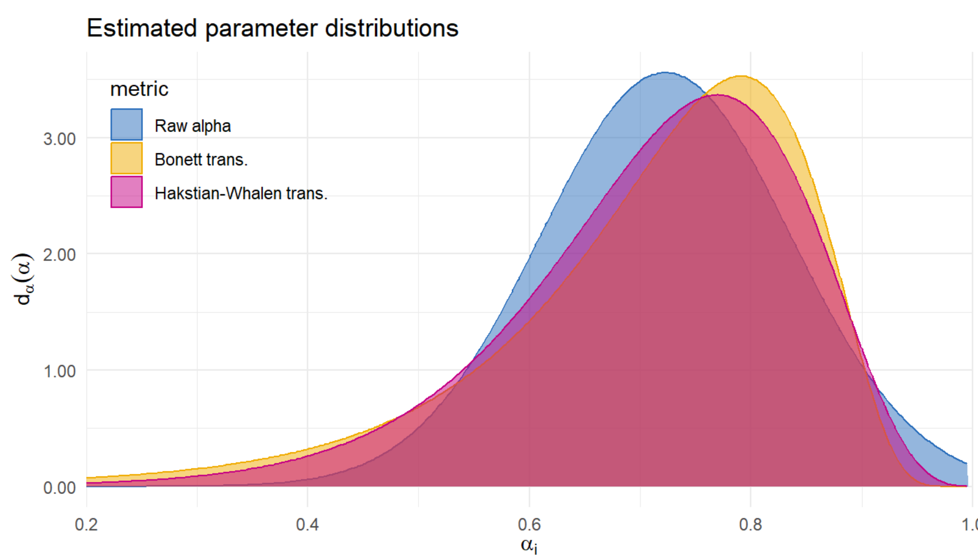


Reliability generalization of MIBS

- Demir and colleagues[20] gathered 33 estimates of internal consistency (Cronbach α) of the Mother-to-Infant Bonding Scale.
- Sample sizes ranging from N_i = 13 to 2251 (median = 177, IQR = 98-260).

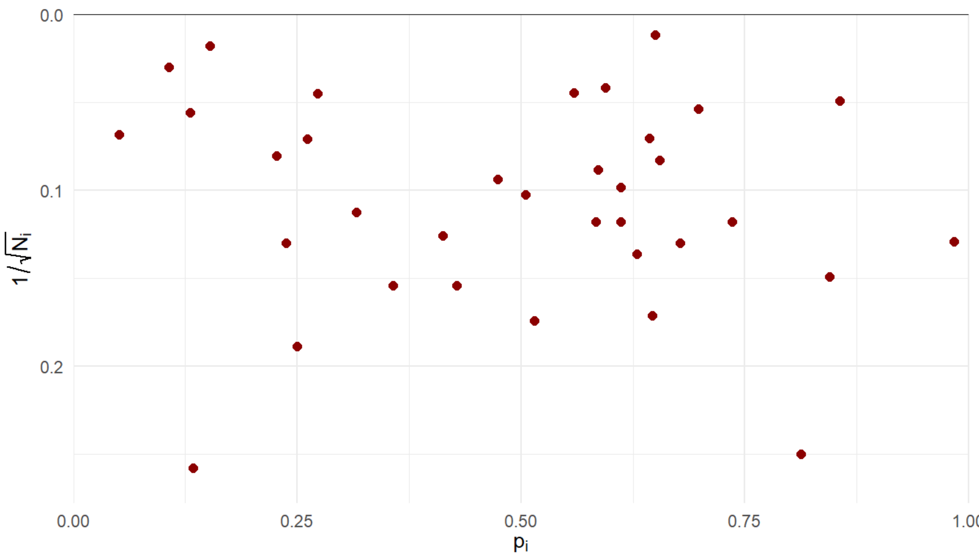


Metric	Est.	95% CI	80% PI	LPD	SE
Raw alpha	0.72	0.68-0.76	0.58-0.87	0.57	0.16
Bonett trans.	0.74	0.69-0.78	0.51-0.86	0.53	0.12
Hakstian-Whalen trans.	0.73	0.68-0.77	0.53-0.86	0.58	0.11



Incidence of olfactory loss in COVID-19 patients

- Hannum and colleagues[21] compiled data on rates of olfactory loss across 35 studies of COVID-19 patients.
- Sample sizes ranging from N_i = 15 to 7178 (median = 95, IQR = 56.5 - 267.5).



- Many different transformations of p_i are used as effect size measures (identity, logit, probit, arcsin-square-root, Freeman-Tukey).
- Could use conventional random effects model or generalized linear mixed model.

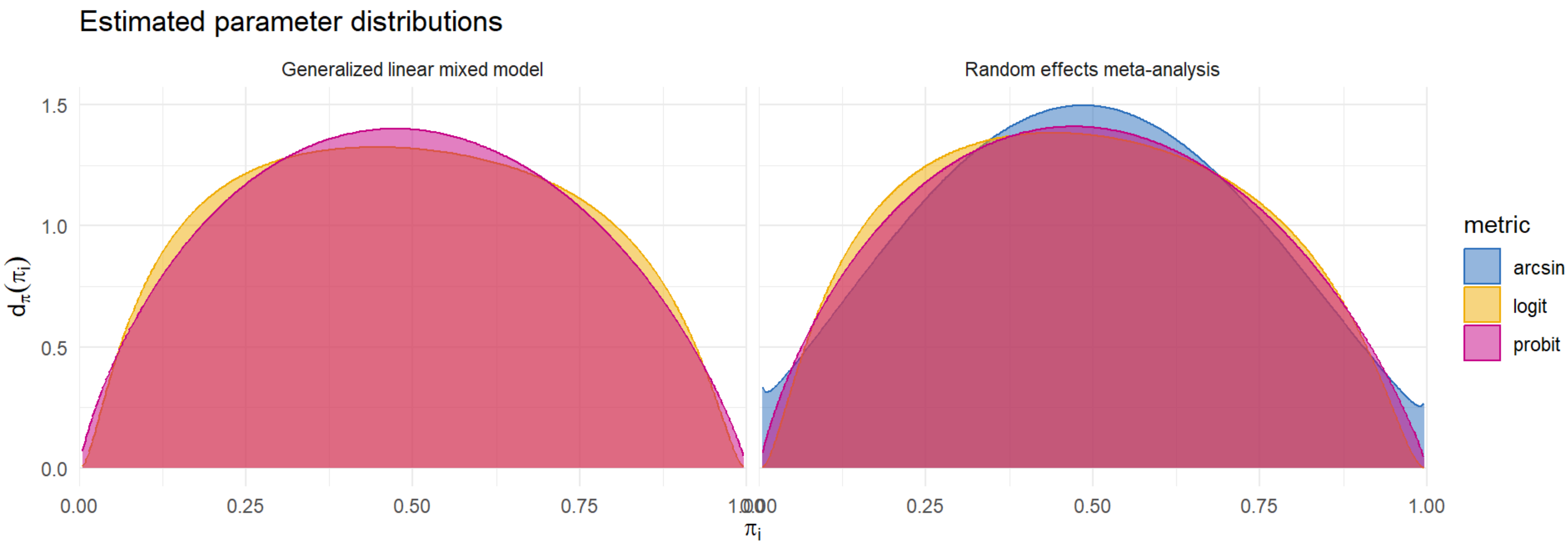
- Which predictive model to use?

$$g(p_i) \dot{\sim} N\left(g(\pi_i),\ \frac{h(\pi_i)}{N_i}\right) \qquad N_i p_i \sim \ Binom\left(N_i,\ \pi_i\right)$$

$$g(\pi_i) \sim \ N\left(\mu_g,\ \tau_g^2\right) \qquad g(\pi_i) \sim \ N\left(\mu_g,\ \tau_g^2\right)$$

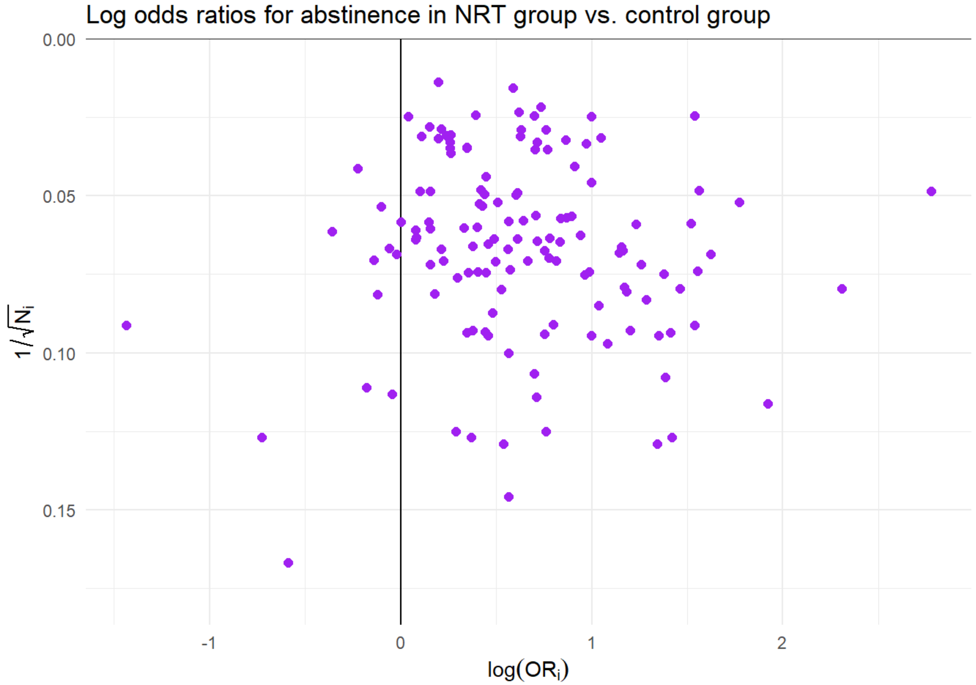
Incidence of olfactory loss in COVID-19 patients

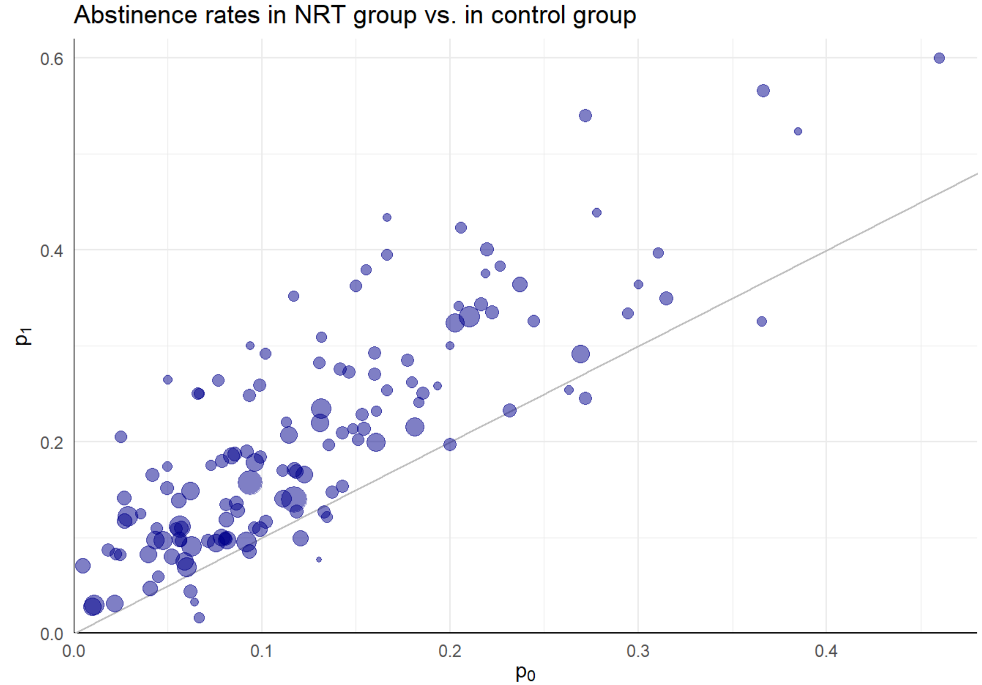
					Normal		Binomial	
Model	Metric	Est.	95% CI	80% PI	LPD	SE	LPD	SE
RE	logit	0.48	0.38-0.58	0.17-0.81	-5.10	0.36	-5.11	0.36
RE	probit	0.49	0.39-0.58	0.17-0.81	-5.18	0.40	-5.18	0.40
RE	arcsin	0.49	0.40-0.58	0.17-0.81	-4.96	0.32	-4.96	0.32
GLMM	logit	0.48	0.38-0.59	0.16-0.82			-5.43	0.55
GLMM	probit	0.49	0.39-0.58	0.17-0.82			-5.24	0.43



Effectiveness of nicotine replacement therapy

- Cochrane Systematic Review of effects of nicotine replacement therapy vs. control on smoking cessation, defined as abstinence at 6+ month follow-up[22].
- Sample sizes ranging from N_i = 36 to 5290 (median = 240.5, IQR = 153.5 - 428.5).



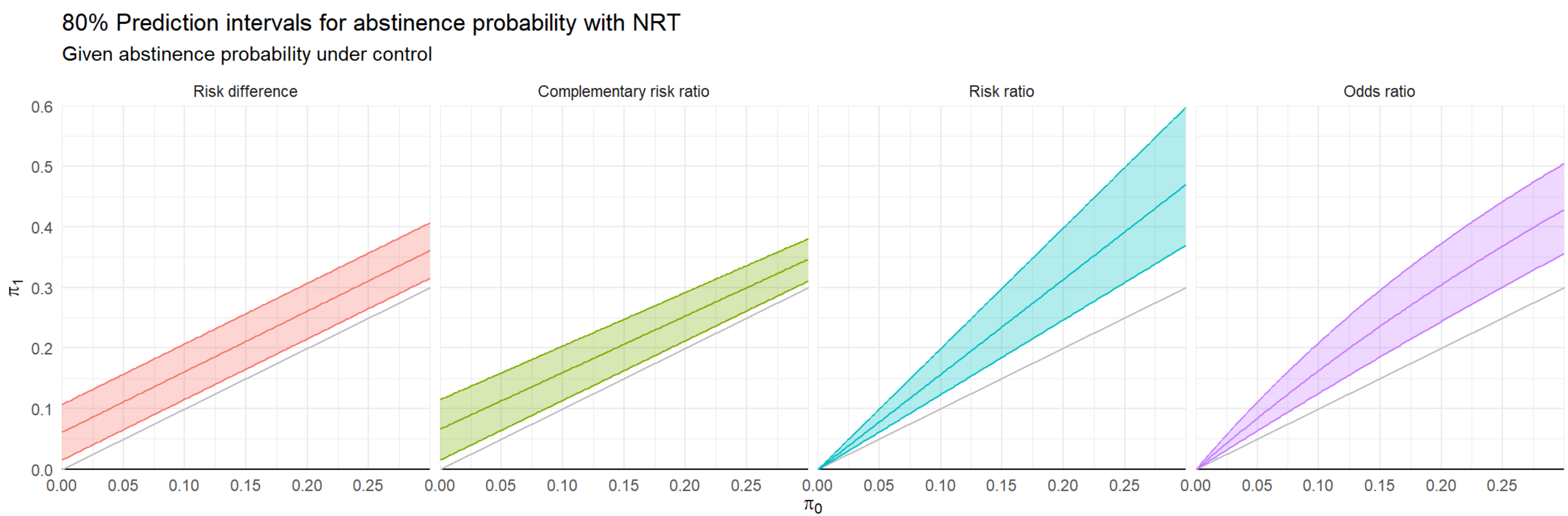


- Multiple possible effect metrics: log odds ratio, log risk ratio, complementary log risk ratio, risk difference
- Alternative models: bivariate meta-analysis[23], bivariate logistic or hypergeometric GLMM[24], baseline risk regression[25–27], etc.

Random effects meta-analysis

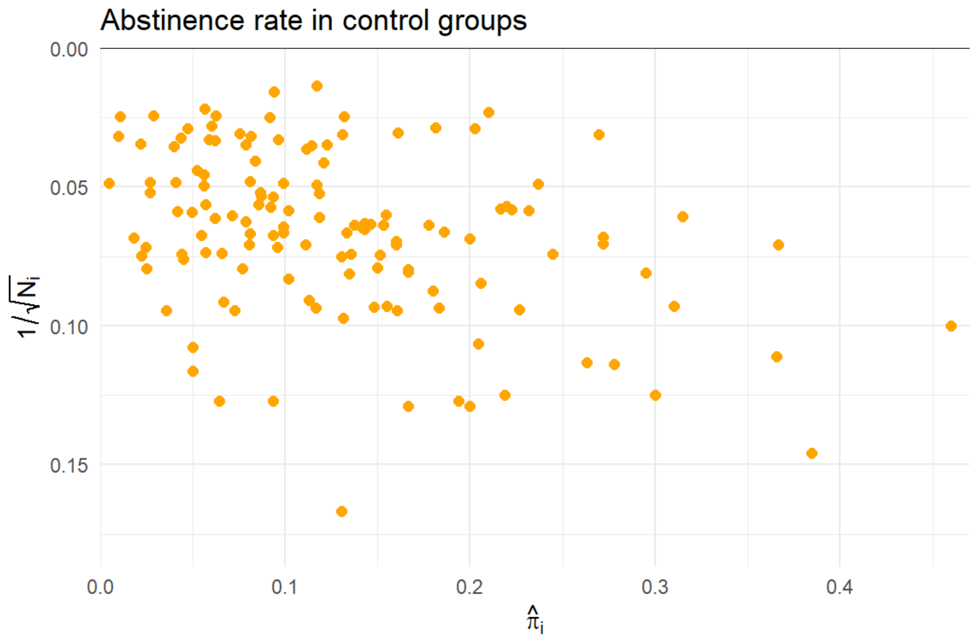
- Difference ES metrics suggest very different implications and different heterogeneity

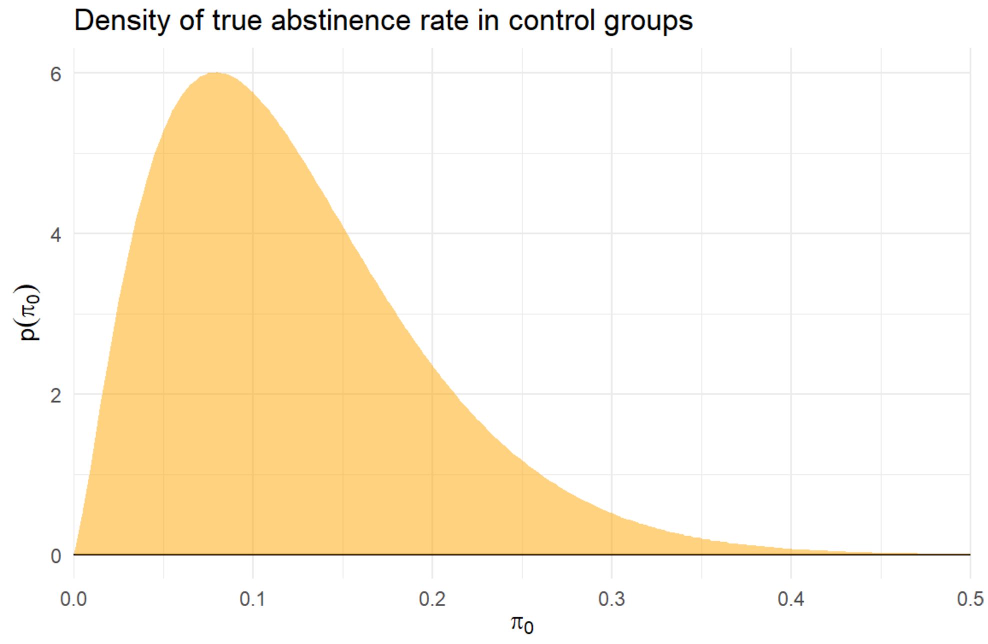
Metric	Est	95% CI	80% PI	I2
Risk difference	0.06	0.05-0.07	0.02-0.11	63.50
Complementary risk ratio	1.07	1.06-1.08	1.02-1.13	65.51
Risk ratio	1.57	1.48-1.66	1.23-1.99	36.88
Odds ratio	1.75	1.63-1.88	1.29-2.38	39.06



Effect metric comparison

- Goal: evaluate predictions of $\hat{\pi}_{0i}, \hat{\pi}_{1i}$ using log-predictive density.
- Conventional RE meta-analysis is a model for $f(\hat{\pi}_{0i}, \hat{\pi}_{1i})$.
- Possible auxiliary models for $\hat{\pi}_{0i}$ or $\hat{\pi}_{1i}$:
  - Random effects meta-analysis/meta-regression
  - Generalized linear mixed model
  - Beta-binomial regression


Abstinence rate in control groups
0.00
0.05
0.10
0.15
$1/\sqrt{N_i}$
0.0
0.1
0.2
0.3
0.4
$\hat{\pi}_i$


Density of true abstinence rate in control groups
6
4
2
0
$p(\pi_0)$
0.0
0.1
0.2
0.3
0.4
0.5
π_0

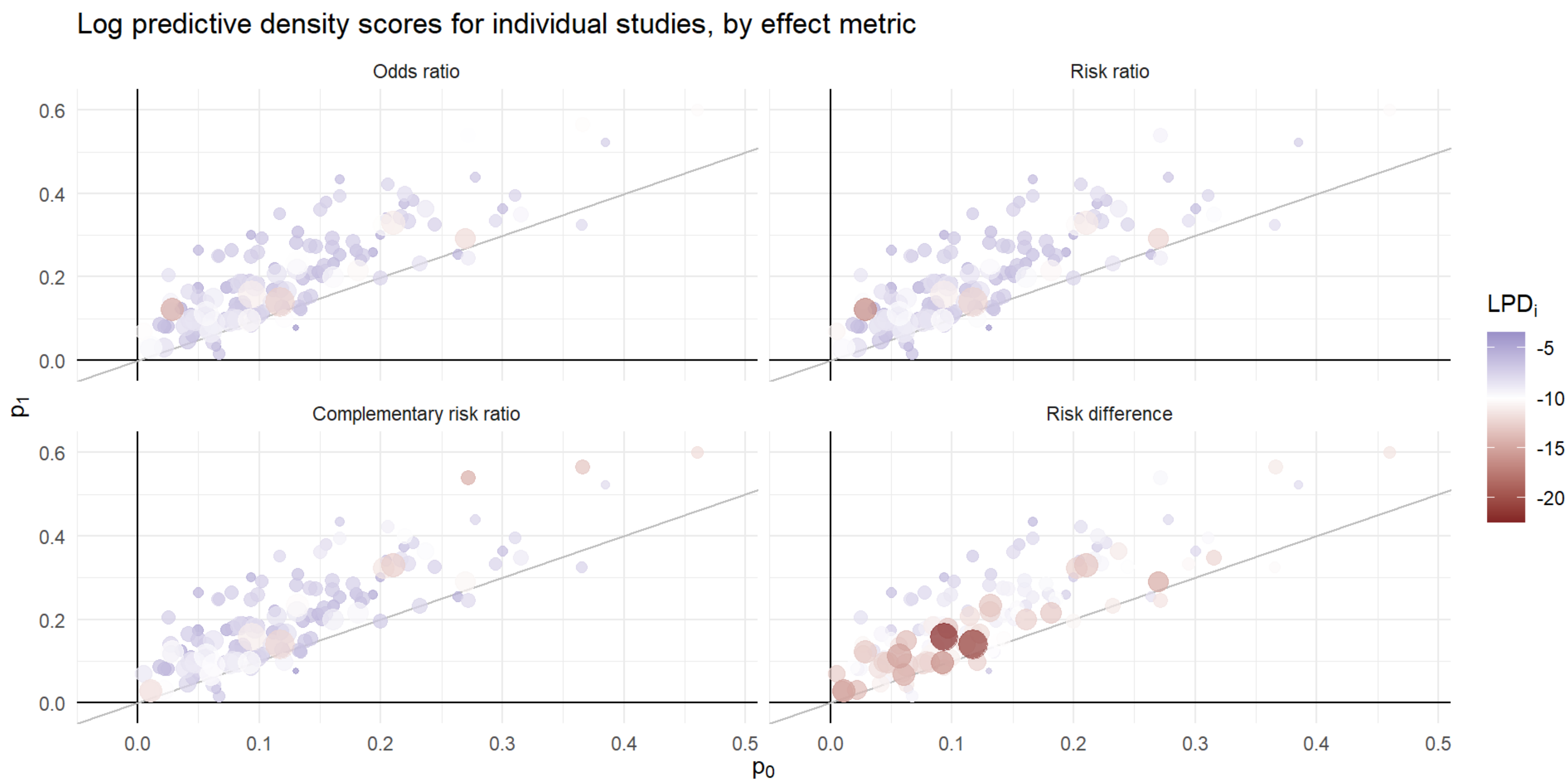
Predictive model

$$
\begin{aligned}
&\text{Auxiliary model:} && \pi_{0i} \sim Beta\,(\alpha,\ \beta) \\
&\text{RE meta-analysis model:} && \theta_i \sim N\left(\mu,\ \tau^2\right) \\
&\text{Observation model:} && N_{0i}\hat{\pi}_{0i} \sim Binom\,(N_{0i},\ \pi_{0i}) \\
& && N_{1i}\hat{\pi}_{1i} \sim Binom\,(N_{0i},\ g(\pi_{0i}, \theta))
\end{aligned}
$$

Metric comparison

Metric	LPD	SE	Diff. vs. OR	SE
Odds ratio	-7.300	0.151		
Risk ratio	-7.342	0.157	-0.041	0.019
Complementary risk ratio	-7.443	0.163	-0.143	0.076
Risk difference	-10.152	0.217	-2.852	0.135

Predictive discrepancies



Discussion

- Effect metric choice is a modeling assumption.
- Predictive fit assessment is relevant and useful for meta-analysis.
  - Log predictive density calculations should be part of meta-analysts' toolkit.
  - Will often require use of auxiliary models.
- Advantages of log predictive density scoring
  - Allows comparison across effect metrics and different forms of models.
  - Auxiliary model building exercise can clarify scientific context.
- Disadvantages and open questions
  - Deshpande and colleagues[28] highlight discrepancies between LPD and other model evaluation metrics.
  - Other predictive scoring rules that may be relevant?
  - Is the joint distribution of $\mathbf{d}_i$ the right focus?

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