# Operationally comparable effect sizes for meta-analysis of single-case research

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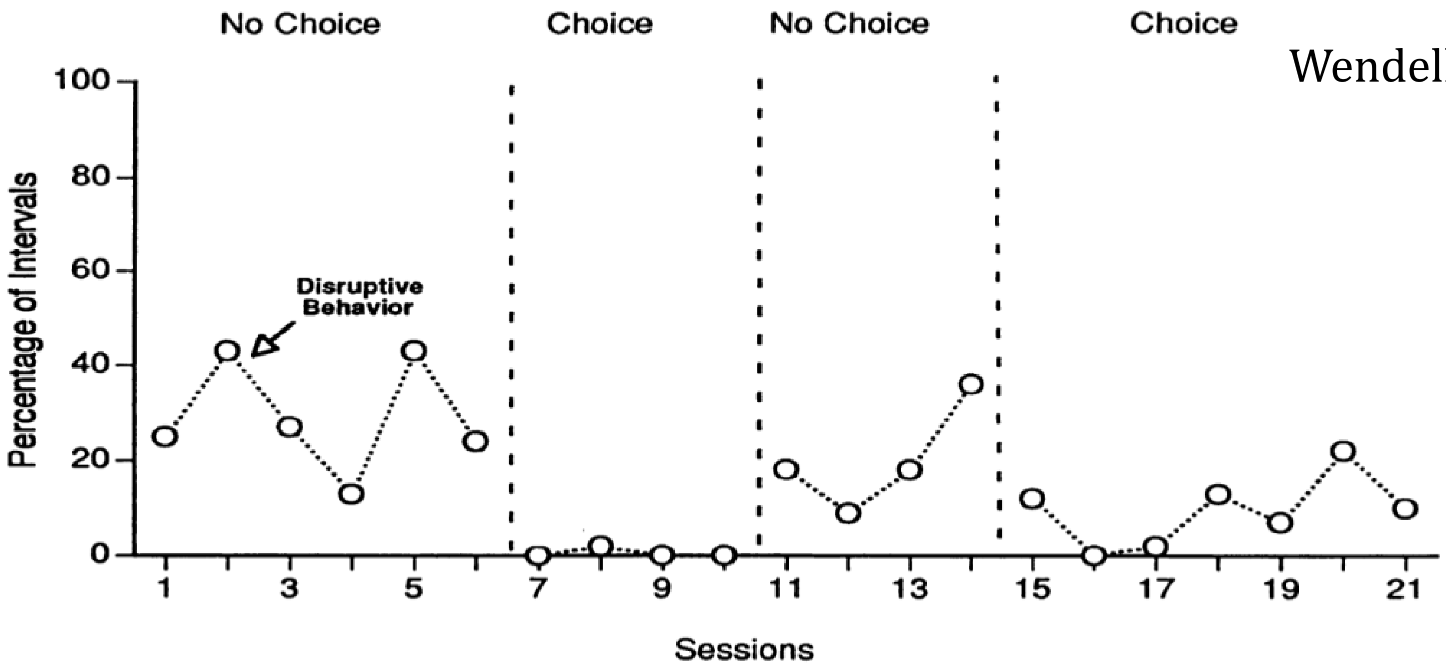
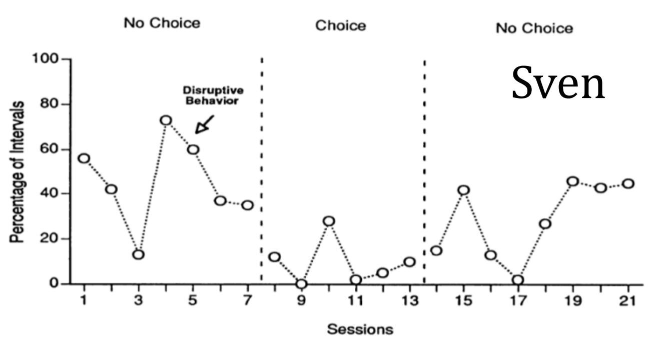
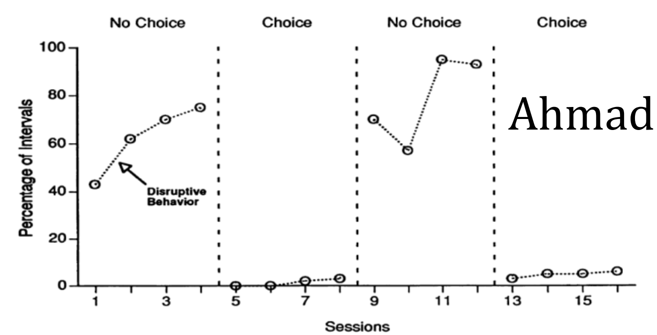
March 7, 2013

# Single Case Designs

Dunlap, et al. (1994). Choice making to promote adaptive behavior for students with emotional and behavioral challenges.



# Meta-analysis of single-case research

- Summarizing results from multiple cases, studies
- Means for identifying evidence-based practices

- Many proposed effect size metrics for single-case designs (Beretvas & Chung, 2008)
  - Computational formulas, without reference to models
  - Mostly focused on standardized mean differences (exceptions: Shadish, Kyse, & Rindskopf, 2012; Sullivan & Shadish, 2013)

# Shogren, et al. (2004)

The effect of choice-making as an intervention for problem behavior

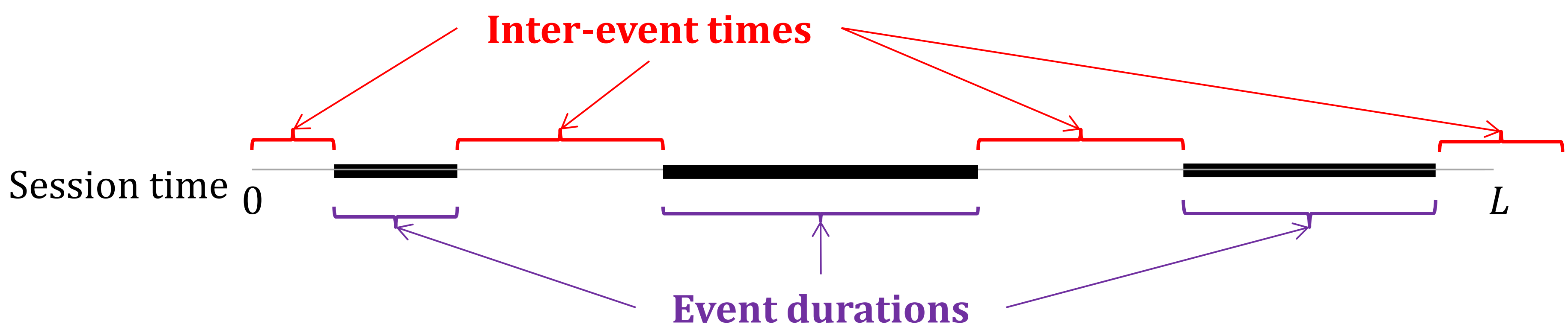
- Meta-analysis containing 13 single-case studies
- 32 unique cases

| Measurement procedure | # Cases |
|---|---:|
| Event counting | 3 |
| Continuous recording | 5 |
| Partial interval recording | 19 |
| Other | 5 |

# Operationally comparable effect sizes

- Separate the definition of effect size metric from the operational details about outcome measurements.
- Parametrically defined
  - Within-session measurement model
  - Between-session model
  - Effect size estimand

# A within-session model for behavior




**Alternating Renewal Process (Rogosa & Ghandour, 1991)**

1. Event durations are identically distributed, with average duration $\mu > 0$.
2. Inter-event times (IETs) are identically distributed, with average IET $\lambda > 0$.
3. Event durations and IETs are all mutually independent.
4. Process is in equilibrium.

# Observation recording procedures

| Procedure | Measured quantity | Expectation under ARP model |
|---|---|---|
| Event counting | Incidence | $\frac{1}{\mu+\lambda}$ |
| Continuous recording | Prevalence | $\frac{\mu}{\mu+\lambda}$ |
| Partial interval recording | Neither prevalence nor incidence | $\frac{\mu}{\mu+\lambda}+\frac{\int_0^P Pr(IET>x)dx}{\mu+\lambda}$ |

# Between-session model

- Baseline phase(s):
  - Independent observations
  - Stable ARP from session to session

$$Y_j \sim \text{Procedure}\left[ARP(\mu_B, \lambda_B)\right]$$

- Treatment phase(s):
  - Independent observations
  - Stable ARP from session to session

$$Y_j \sim \text{Procedure}\left[ARP(\mu_T, \lambda_T)\right]$$

# The prevalence ratio

- The prevalence ratio:

$$\Omega = \frac{\mu^T / \left(\mu^T + \lambda^T\right)}{\mu^B / \left(\mu^B + \lambda^B\right)}$$

- Why?
  - Prevalence is often most practically relevant dimension.
  - Ratio captures how single-case researchers talk about their results.
  - Empirical fit.

- Confidence intervals, meta-analysis on natural log scale.

$$\omega = \log\left(\frac{\mu^T}{\mu^T + \lambda^T}\right) - \log\left(\frac{\mu^B}{\mu^B + \lambda^B}\right)$$

# Estimating the prevalence ratio

- Continuous recording
  - Response ratios (Hedges, Gurevitch, & Curtis, 1998)
  - Generalized linear models
- Event counting
  - Incidence ratio equal to prevalence ratio if average event duration does not change ($\mu_{\mathrm{B}} = \mu_{\mathrm{T}}$)
- Partial interval data
  - Need to invoke additional, rather strong assumptions even to get bounds on prevalence ratio
  - For example: Assuming $\mu^B, \mu^T > \mu_{min}$ for known $\mu_{min}$ implies a bound on the prevalence ratio.

# Conclusion

- Limit scope to a specific class of outcomes (directly observed behavior).
- Use a model to
  - Address comparability of different outcome measurement procedures.
  - Separate effect size definition from estimation procedures.
- Emphasize assumptions that justify estimation strategy.

- Still need to address comparability with effect sizes from between-subjects designs
  (Shadish, Hedges, & Rindskopf, 2008; Hedges, Pustejovsky, & Shadish, 2012)

# References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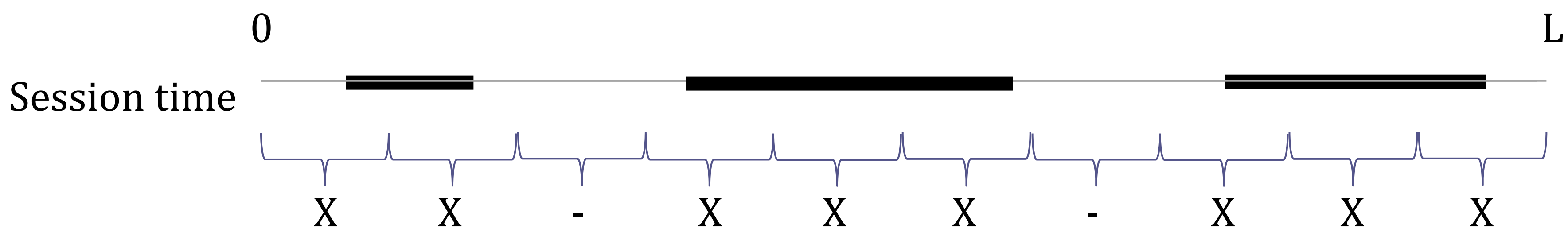
# Single-case designs

- Repeated measurements, often via direct observation of behaviors
- Comparison of outcomes pre/post introduction of a treatment
- Replication across a small sample of cases.

# Partial interval recording




1. Divide session into *K* short intervals, each of length *P*.
2. During each interval, note whether behavior occurs at all.
3. Calculate proportion of intervals where behavior occurs:

$Y$ = (# Intervals with behavior) / $K$.

# Possible effect sizes for free-operant behavior

| | |
|---|---|
| $\dfrac{\mu^T}{\mu^B}$ | Duration Ratio |
| $\dfrac{\lambda^T}{\lambda^B}$ | Inter-Event Time Ratio |
| $\dfrac{\mu^B + \lambda^B}{\mu^T + \lambda^T}$ | Incidence Ratio |
| $\dfrac{\mu^T / \left(\mu^T + \lambda^T\right)}{\mu^B / \left(\mu^B + \lambda^B\right)}$ | Prevalence Ratio |
| $\dfrac{\mu^T / \lambda^T}{\mu^B / \lambda^B}$ | Prevalence Odds Ratio |

# Outcomes in single-case research

| Outcome | % of Studies |
|---|---|
| Free-operant behavior | 56 |
| Restricted-operant behavior | 41 |
| Academic | 8 |
| Physiological/psychological | 6 |
| Other | 3 |

N = 122 single-case studies published in 2008, as identified by Shadish & Sullivan (2011).

- **Restricted-operant** behavior occurs in response to a specific stimulus, often controlled by the investigator.
- **Free-operant** behavior can occur at any time, without prompting or restriction by the investigator (e.g., physical aggression, motor stereotypy, smiling, slouching).

# Measurement procedures for free-operant behavior

| Recording procedure | % of Studies |
|---|---:|
| Event counting | 60 |
| Interval recording | 19 |
| Continuous recording | 10 |
| Momentary time sampling | 7 |
| Other | 16 |

N = 68 single-case studies measuring free-operant behavior, a subset of all 122 studies published in 2008, as identified by Shadish & Sullivan (2011). Characteristics of single-case designs used to assess intervention effects in 2008. *Behavior Research Methods*, *43*(4), 971–80.

# Effect size estimation: Continuous recording

- A basic moment estimator:

$$\hat{\omega} = \log\left(\bar{y}_T\right) - \log\left(\bar{y}_B\right)$$

$$\bar{y}_B = \frac{1}{n^B}\sum_{j=1}^{n^B} Y_j\left(1 - Trt_j\right) \qquad \bar{y}_T = \frac{1}{n^T}\sum_{j=n^B+1}^{n^B+n^T} Y_j Trt_j$$

- Its approximate variance:

$$Var\left(\hat{\omega}\right) \approx \frac{s_T^2}{n_T\left(\bar{y}_T\right)^2} + \frac{s_B^2}{n_B\left(\bar{y}_B\right)^2}$$

$$s_B^2 = \frac{1}{n^B - 1}\sum_{j=1}^{n^B}\left(1 - Trt_j\right)\left(Y_j - \bar{y}_B\right)^2 \qquad s_T^2 = \frac{1}{n^T - 1}\sum_{j=n^B+1}^{n^B+n^T} Trt_j\left(Y_j - \bar{y}_T\right)^2$$

# Partial interval data: Analysis strategies

- Strategy 1:
  - Assume that $\mu^B$, $\mu^T > \mu_{min}$ for known $\mu_{min}$.
  - Estimate bounds on the true prevalence ratio.

- Strategy 2:
  - Assume that $\mu^B = \mu^T$
  - Assume that inter-event times are exponentially distributed.
  - Estimate bounds on true prevalence ratio ("sensitivity analysis").

- Strategy 3:
  - Follow strategy 2, but for known $\mu^* = \mu^B = \mu^T$ .
  - This leads to a point estimate for the prevalence ratio.

# Partial interval data: Strategy 1

- Pick a value $\mu_{min}$ where you are certain that $\mu^B, \mu^T > \mu_{min}$ .
- Then, under ARP,

$$\Omega^L \leq \Omega \leq \Omega^U$$

where

$$\Omega^L \equiv \frac{E\left(Y^T\right)}{E\left(Y^B\right)} \times \left(\frac{\mu_{min}}{\mu_{min} + P}\right) \qquad \Omega^U \equiv \frac{E\left(Y^T\right)}{E\left(Y^B\right)} \times \left(\frac{\mu_{min} + P}{\mu_{min}}\right)$$

$Y^B$ outcome in baseline phase
$Y^T$ outcome in treatment phase

# Partial interval data: Strategy 1 (cont.)

- Estimate the bounds with sample means.

$$\hat{\Omega}^L \equiv \frac{\bar{y}_T}{\bar{y}_B} \times \left( \frac{\mu_{min}}{\mu_{min} + P} \right) \qquad \hat{\Omega}^U \equiv \frac{\bar{y}_T}{\bar{y}_B} \times \left( \frac{\mu_{min} + P}{\mu_{min}} \right)$$

$\bar{y}_B$ sample mean in baseline phase, $\bar{y}_T$ sample mean in treatment phase

- With approximate variance (on log-scale)

$$Var\left(\log \hat{\Omega}^L\right) = Var\left(\log \hat{\Omega}^U\right) \approx \frac{s_T^2}{n_T\left(\bar{y}_T\right)^2} + \frac{s_B^2}{n_B\left(\bar{y}_B\right)^2}$$

$s_B^2$ sample variance in baseline phase, $s_T^2$ sample variance in treatment phase

$n_B$ observations in baseline phase, $n_T$ observations in treatment phase

# Partial interval data: Strategy 2

- Assume that IETs are exponentially distributed.
- Assume that $\mu^B = \mu^T$.
- If E($Y^T$) < E($Y^B$) then $\omega^L \leq \omega \leq \omega^U$

$$\omega^L = \ln\left[-\ln E\left(1-Y^B\right)\right] - \ln\left[-\ln E\left(1-Y^T\right)\right]$$

$$\omega^U = \ln\left[E\left(Y^T\right)\right] - \ln\left[E\left(Y^B\right)\right]$$

- Estimate the bounds with sample means.

$$\hat{\omega}^L = \ln\left[-\ln\left(1-\bar{y}_B\right)\right] - \ln\left[-\ln\left(1-\bar{y}_T\right)\right]$$

$$\hat{\omega}^U = \ln\left(\bar{y}_T\right) - \ln\left(\bar{y}_B\right)$$

# Partial interval data: Strategy 3

Assumptions:

1. IETs are exponentially distributed.
2. Average duration is constant across phases: $\mu^B = \mu^T$.
3. Assume that $\mu^B = \mu^T = \mu^*$, for some known $\mu^*$.

# Partial interval data: Strategy 3 (cont.)

- Find estimates for $\lambda^B$ and $\lambda^T$ by solving

$$\bar{y}_B = 1 - \hat{\lambda}^B e^{-P/\hat{\lambda}^B} / \left(\mu^* + \hat{\lambda}^B\right) \qquad \bar{y}_T = 1 - \hat{\lambda}^T e^{-P/\hat{\lambda}^T} / \left(\mu^* + \hat{\lambda}^T\right)$$

- Estimate $\Omega$ with

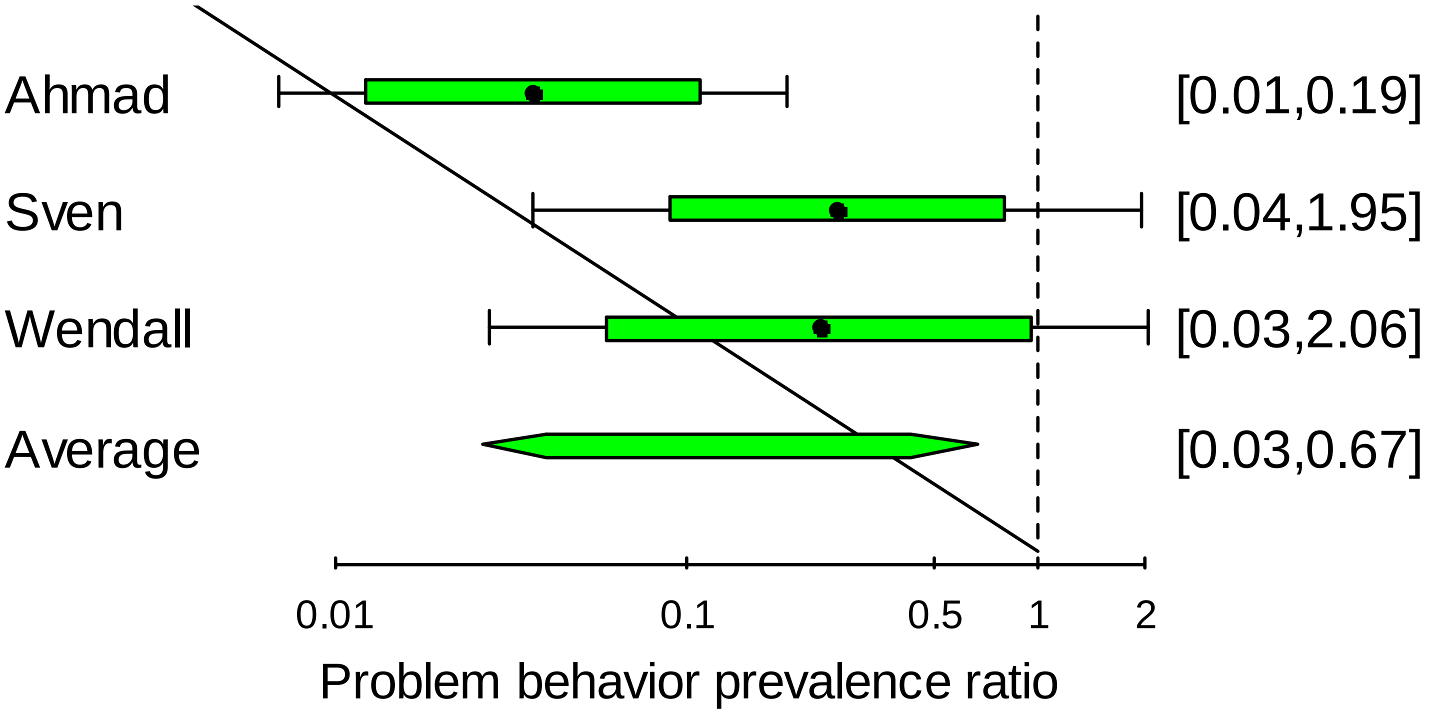
$$\hat{\Omega} = \frac{\mu^* / \left(\mu^* + \hat{\lambda}^T\right)}{\mu^* / \left(\mu^* + \hat{\lambda}^B\right)}$$

$$Var\left(\log \Omega\right) \approx \sum_{p=B,T} \frac{\left(\hat{\lambda}^p\right)^4 s_p^2}{\left(1 - \bar{y}_p\right)^2 \left[\mu^* \hat{\lambda}^p + P\left(\mu^* + \hat{\lambda}^p\right)\right]^2}$$

# Dunlap, et al. (1994): Strategy 1

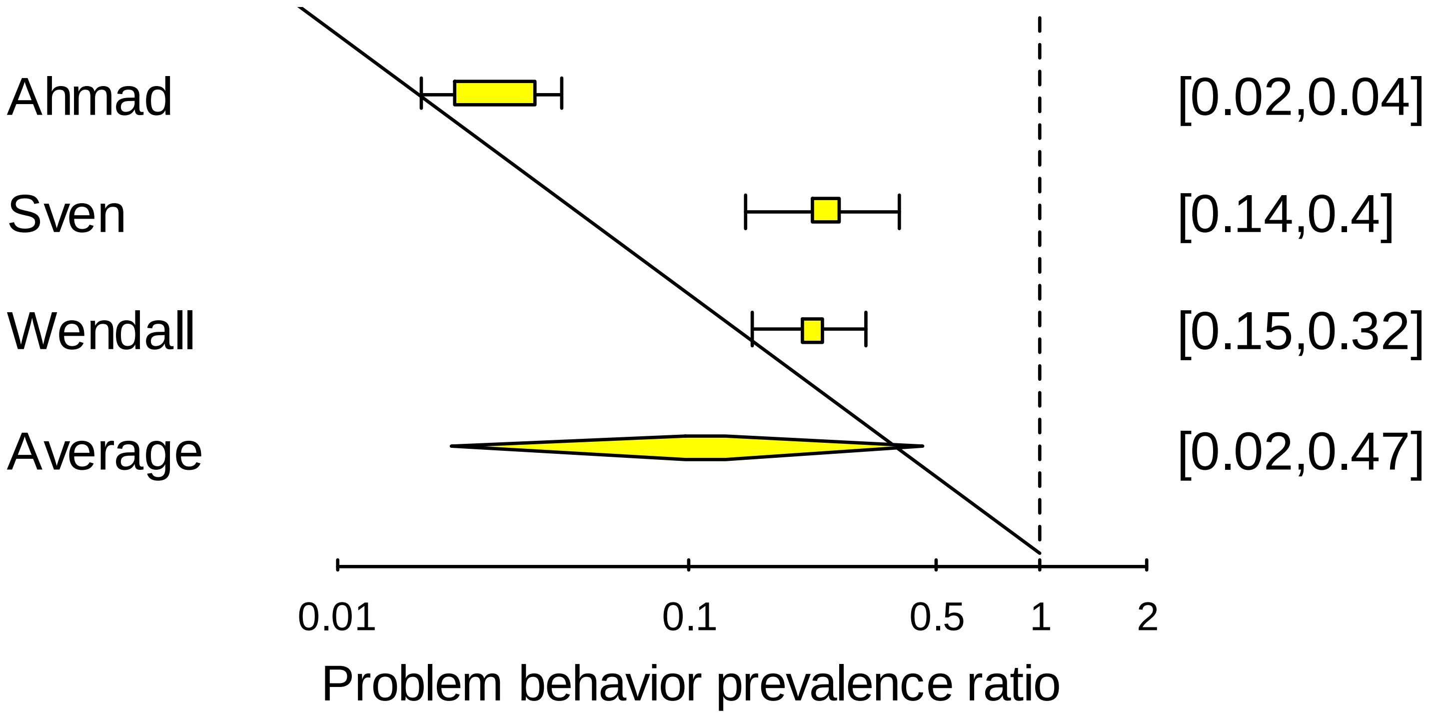
Choice making to promote adaptive behavior for students with emotional and behavioral challenges.

$\mu_{min} = 5\ s$



# Dunlap, et al. (1994): Strategy 2

Choice making to promote adaptive behavior for students with emotional and behavioral challenges.



# Shogren (2004) meta-analysis

The effect of choice-making as an intervention for problem behavior.